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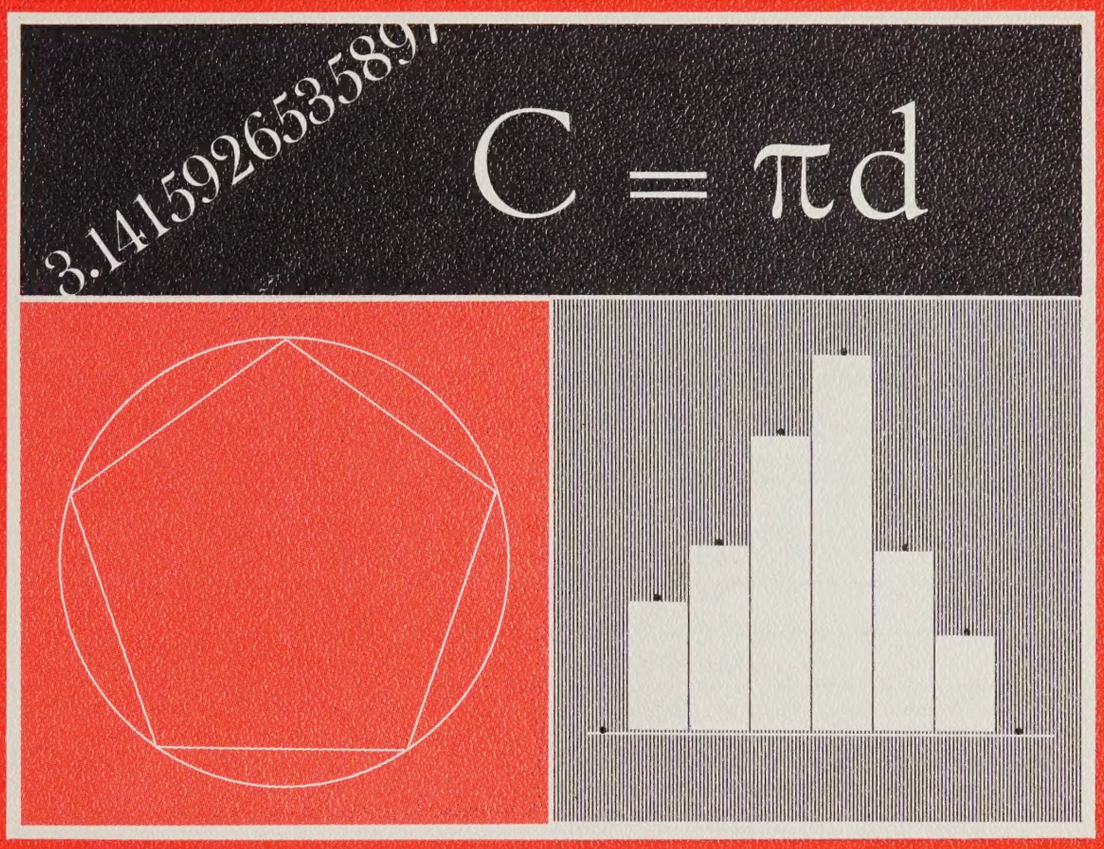

MATHEMATICS

grade 8

EDUCATIONAL TELEVISION SERIES 1967-68

Educational Television Branch
ONTARIO DEPARTMENT OF EDUCATION
1670 Bayview Avenue/Toronto 17

program schedule

MATHEMATICS
grade 8

22 programs of 30 minutes each

PROGRAM NO.	CBC 9:30	CBOT 9:30	CHCH 9:30
1	Oct. 18	Same as CBC	Oct. 27
2	Oct. 25	" "	Nov. 3
3	Nov. 1	" "	Nov. 10
4	Nov. 8	" "	Nov. 17
5	Nov. 15	" "	Nov. 24
6	Nov. 22	" "	Dec. 1
7	Nov. 29	" "	Dec. 8
8	Dec. 6	" "	Dec. 15
9	Dec. 13	" "	Jan. 12
10	Dec. 20	" "	Jan. 19
11	Jan. 10	" "	Jan. 26
12	Jan. 17	" "	Feb. 2
13	Jan. 24	" "	Feb. 9
14	Jan. 31	" "	Feb. 16
15	Feb. 7	" "	Feb. 23
16	Feb. 14	" "	Mar. 1
17	Feb. 21	" "	Mar. 8
18	Feb. 28	" "	Mar. 15
19	Mar. 6	" "	Mar. 29
20	Mar. 13	" "	Apr. 5
21	Mar. 27	" "	Apr. 19
22	Apr. 3	" "	Apr. 26

MATHEMATICS—Grade 8

A series of 22 programs of 30 minutes each

Divided into 2 sections: Geometry—programs 1 to 17
Statistics and Probability—programs 18 to 22

contents of geometric concepts

Program Schedule		inside front cover
General Introduction		2
Program one	*Geometry—The science of size and shape*	3
Program two	*Plane figures in geometry*	4
Program three	*Solids and surfaces*	6
Program four	*Geometry involved in model construction*	8
Program five	*To construct an angle equal to a given angle*	10
Program six	*To bisect a line segment*	11
Program seven	*To bisect an angle—To construct a line perpendicular to a line to a point in a line*	13
Program eight	*To draw a line perpendicular to a line from a point outside a line*	15
Program nine	*To construct a line parallel to a given line*	17
Program ten	*Perimeter and circumference*	19
Program eleven	*Regions and area*	20
Program twelve	*Units for measurement of area*	22
Program thirteen	*Developing a formula for calculating area*	23
Program fourteen	*The Pythagorean theorem*	26
Program fifteen	*Surface area of three dimensional figures*	28
Program sixteen	*Introduction to volume*	30
Program seventeen	*Volume of prisms and cylinders*	31

contents of statistics and probability

Program eighteen	*Statistics*	33
Program nineteen	*Statistics*	34
Program twenty	*Statistics*	36
Program twenty-one	*Probability*	37
Program twenty-two	*Probability*	38
Optimum Viewing Conditions		inside back cover

Subject Supervisor:

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G. A. SCROGGIE, Assistant Superintendent, Curriculum Division

Teachers Guide prepared and written by:

for programs 1-17

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for programs 18-22

JAMES B. BLUE, Principal, Western School, Windsor

general introduction

This series of 22 programs is based on the course of study for grade 8 Mathematics (see Curriculum Bulletin 1.12B, unit six: Geometry and Measurement; and unit seven: Statistics and Probability.) The teachers guide supplies essential information.

As each program lends itself to approximately one follow-up class period, it is suggested that teachers plan their teaching of units six and seven to cover the 22 weeks of the telecast. One period for the film and subsequent discussion, and, where possible, one period for the kind of follow-up activities suggested in the guide would assure the best use of these programs. It is essential that students be equipped with the materials indicated in the guide as they will be expected to participate during the telecasts.

In several programs, the students are given the opportunity to reply orally to questions posed by the speaker. The individual program outline in the teachers guide will make note of this if the technique is to be applied. It is suggested that the teacher ask any pupil to answer the question. In some programs, a problem is left for the students to solve after the telecast. It is important that this be completed.

The programs are designed not only to stimulate student interest in and understanding of Geometry but also to present the teacher with some useful ideas concerning presentation of lesson material to students. The intuitive approach is evident as students discover and analyze situations and make conjectures which are often tested by measurement. Such experience should be beneficial to the student when confronted in later years with mathematical problems that require formal proof. Grade 7 work is reviewed briefly at the beginning of the series so that students with little or no background can get the full benefit of the programs.

program one

GEOMETRY—THE SCIENCE OF SIZE AND SHAPE

The first program in this series of telecasts on "Geometric Concepts" treats the subject as an intuitive study of shape and size. An attempt has been made to take the television camera outside the classroom to show many examples of geometric objects from the real world. In this way it is hoped that students will become more observant and aware of geometric structures in their environment and realize that geometry plays an important part in their lives.

PROGRAM CONTENT

The program begins by showing some of the shapes in the Universe, eventually focussing on our planet Earth. This is followed by film sequences showing some of nature's geometric handwork in plant and animal life.

The ingenuity of Man is then recognized, as the camera portrays some very primitive, man-made structures followed by some famous and magnificent ones, which illustrate the deliberate use of geometric structure, particularly in the field of architecture. Some buildings are visited a second time, and in order to stress their basic three-dimensional geometric shapes, fundamental models of these shapes are superimposed on them. As these models are analysed, it is pointed out that their classifications depend upon the shapes of their faces.

Then, the faces are analysed and it is observed that they in turn depend upon their boundaries which are "simple closed curves". This topic "simple closed curves" receives some useful discussion.

The program goes on to suggest that three-dimensional figures can be envisaged in at least three ways. For example, the cube is shown first as a solid, then as a shell where just the surfaces are considered, and finally as a skeleton where just the edges are shown. Students are then encouraged to look for three dimensional shapes in their community environment and to try to classify them into sets of prisms, pyramids, cones, cylinders and spheres.

PREPARATION FOR TELECAST

Each classroom could have a set of geometric models containing prisms, pyramids, cones, cylinders and spheres. The opportunity to see, handle, and discuss these before the program would benefit the student.

SUGGESTED POST-VIEWING ACTIVITIES

1. Students could be given ample opportunity to handle three-dimensional objects including prisms, pyramids, cones, cylinders and spheres and to discuss in their own words their respective attributes. In program 3 these will be dealt with more fully after a discussion of plane figures in program 2.
2. Students could be encouraged to look for examples of basic shapes in the world around them, and to bring examples or pictures of some of these to school. Such items not only add excitement to the class but also give an interesting décor to the classroom. Many of these objects can then be sub-classified as solids, shells or skeletons. For example:

a. A fence post is an example of a solid cylindrical shape.
b. A funnel is an example of a cone where the shell is of use.
c. A hydro tower may be considered a skeleton and is made up of prisms and pyramids.

Bibliography
Adventure in Geometry, *Ravielli*, Macmillan

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program two

PLANE FIGURES IN GEOMETRY

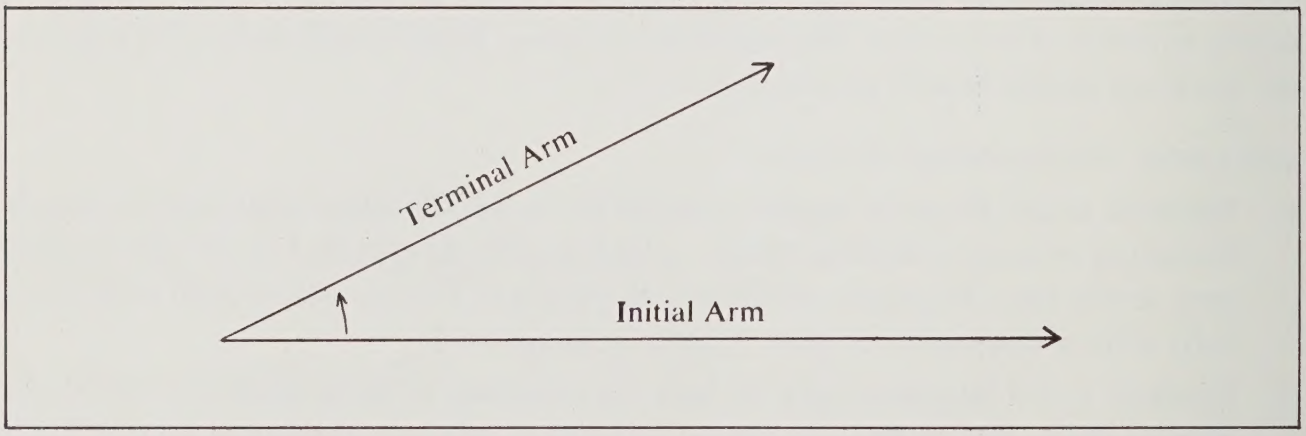
The program begins by acknowledging the premise that we live in a three-dimensional concrete world. However, the abstract ideas of plane, point, and sets of points play a fundamental part in analysing and describing shapes within this world.

PROGRAM CONTENT

The concepts of point, line, ray, segment, and angle are reviewed in the early part of the program. The angle is discussed in two ways: first, as a plane figure formed by two rays with a common end point and secondly, the measure of the angle is described as the amount of rotation of a ray about its end point, from an initial position to a final or terminal position.

Acute, right, obtuse and straight angles are then defined, followed by a description of the protractor and its use in measuring angles. Examples in constructing and measuring angles are given and the student audience is asked to participate.

Simple closed curves are mentioned again and the polygon is defined as a simple closed curve composed only of line segments. Then the triangle, quadrilateral, pentagon, hexagon, octagon and n-gon are defined as polygons with three, four, five, six, eight, and "n" sides respectively. The program closes as on-camera students attempt to find a method of determining the sum of the interior angles in any polygon. The student audience is asked to participate at the appropriate time by filling in a prepared table.



This diagram indicates the rotation from the initial arm or position to the terminal arm or position.

This program provides an opportunity for students to give oral answers to questions posed by the narrator.

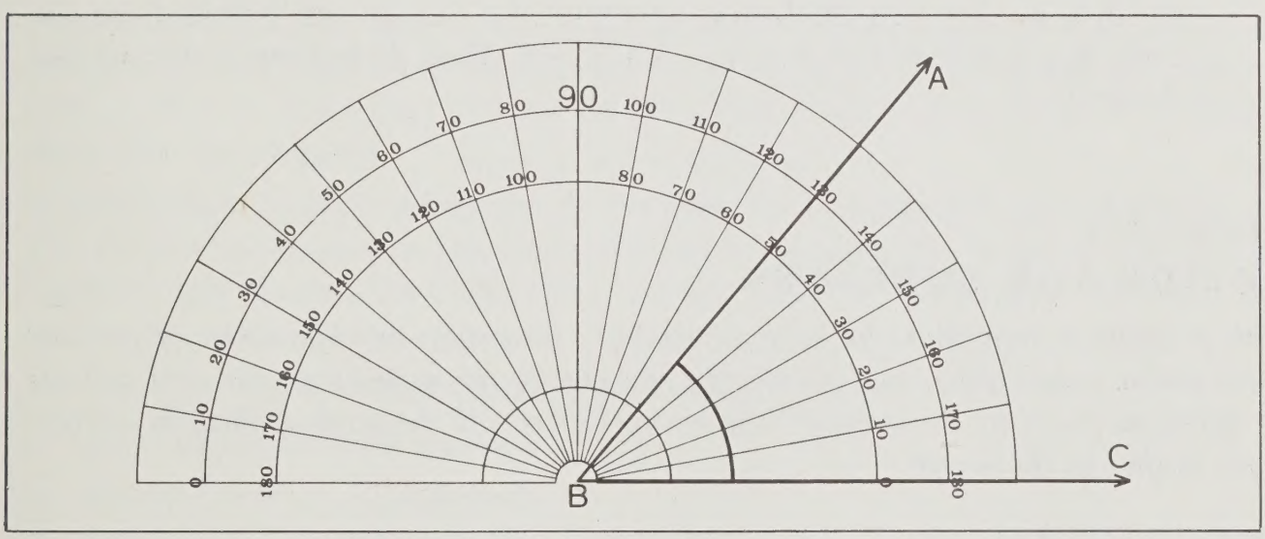
PREPARATION FOR TELECAST

Name of Polygon	*No. of Sides*	*Angle Sum in Right Angles*
Triangle		
Quadrilateral		
Pentagon		

In order to participate fully in this program, each student should have the following materials: pencil, ruler, protractor and workbook with the following table ruled in it.

SUGGESTED POST-VIEWING ACTIVITIES

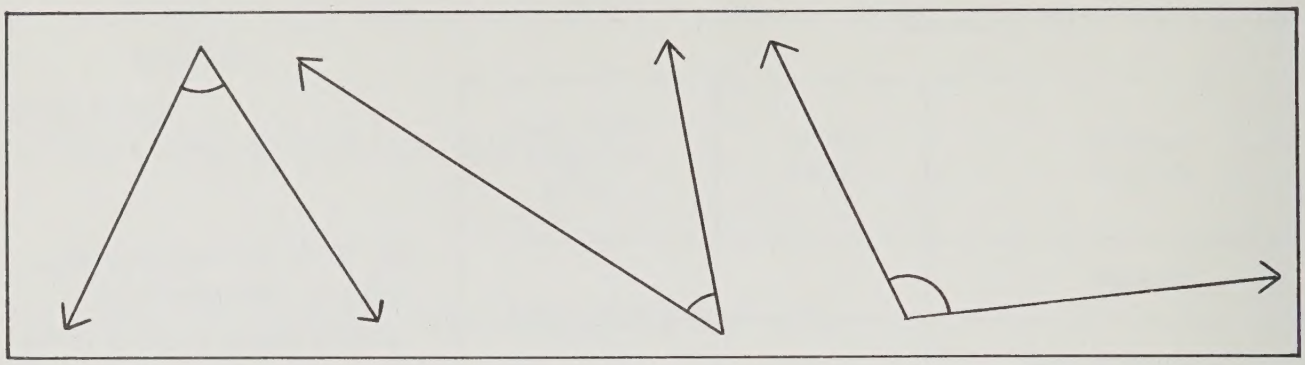
1. Students could be encouraged to discuss various definitions and points of interest from the telecast.
2. The approach taken in the introduction to the use of a protractor is dependent upon the use of rays as arms of the angle. The protractor may be placed on either ray making it the "base" ray. The arrow of the base ray will always point to zero on the scale from which the reading should be made.



Here, BC is used as the base ray, and it passes through zero on the INNER SCALE.
∴ $\angle ABC = 50°$

Students should now place the protractor along ray BA as base ray and measure $\angle ABC$.

Further practice with this method can be obtained by changing the position of the angle, but always indicating the arc of rotation as follows:



3. The table which was set up to find the sum of the angles in a polygon could be extended. Incidentally, the angle sum in an n-gon can be expressed as 2(n-2) right angles.
4. An exercise such as the following could be attempted and discussed:
   a. How many lines can be drawn through one point?
   b. How many line segments can be drawn to join two points?
   c. How many segments can be drawn to join three (non-collinear) points?
   d. How many segments can be drawn to join four (non-collinear) points A, B, C and D?
   (There are six: AB, AC, AD, BC, BD, CD. Note the order in which they are named).
   e. How many segments can be drawn to join the five points A, B, C, D and E? (There are ten, namely AB, AC, AD, AE, BC, BD, BE, CD, CE, DE.)
   f. How many segments can be drawn to join 10 points? (By noting the pattern which is emerging in the former examples, we can get our answer from the series $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$. Thus, 45 line segments could be drawn.)

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program three

SOLIDS AND SURFACES

This program is designed to increase the student's awareness and knowledge of two and three dimensional figures, and provides the opportunity for students to give oral answers to questions posed by the speaker. The answers given will be verified when the correct reply flashes on the screen.

PROGRAM CONTENT

The program opens by defining regular polygons as polygons with all sides and all angles equal.

Then triangles are classified according to sides (equilateral, isosceles, scalene) and according to the largest angle (acute-angled, right-angled and obtuse-angled).

Quadrilaterals are also classified into sets of trapeziods, parallelograms, rectangles and squares.

The circle is defined as the set of all points equidistant from a given point; this is followed by descriptions of parts associated with the circle such as centre, minor, and major arcs, radius and diameter.

The next portion of the program is devoted to description, definition, and discussion of three-dimensional figures. If a three-dimensional figure has plane faces which are all polygons, it is called a polyhedron. If the faces of the polyhedron are all congruent regular polygons, it is called a regular polyhedron. Only five of these exist. The tetrahedron is the simplest and has four equilateral triangles for faces. The hexahedron is commonly known as the cube and has six squares for faces. The octahedron and icosahedron have eight and twenty faces respectively, all being equilateral triangles. The dodecahedron has twelve regular pentagons for faces.

For a polyhedron to be a prism, the top and bottom, which are called bases, must be congruent and parallel polygons. The other faces or lateral surfaces must be parallelograms. Another common polyhedron is the pyramid. Its lateral surfaces are triangles with a common vertex while its base can be any polygon. Prisms and pyramids are classified and named according to their bases. Thus there are triangular prisms and triangular pyramids as well as hexagonal prisms and square pyramids.

Cones, cylinders, and spheres are discussed briefly.

A cone is then sliced in several ways and the circle, ellipse, and parabola are highlighted. Methods of drawing each of these curves are then illustrated, and the audience is encouraged to participate in the drawing of a parabola. The program closes as students practise the art of curve stitching.

PREPARATION FOR TELECAST

In order to participate fully in this program, each student should have a ruler, compasses, and a notebook on his desk. It would be helpful if models of prisms, pyramids, cones, cylinders, and spheres were available so the students could handle and discuss them after the program.

If Bristol board, needles and thread are available, students can do some curve stitching. The material developed in this program may be too lengthy for some classes. The teacher might prepare his students by reviewing, before the telecast, the following items:

1. Classification of triangles according to angles or sides.
2. Parts associated with a circle.
3. Classification of quadrilaterals.
   (see program content, first four paragraphs).

SUGGESTED POST-VIEWING ACTIVITIES

1. Students could be encouraged to discuss the definition of the various polygons and solids, and thus review the essential features in each classification.

2. Plasticene models of solids could be made and then shapes noted as they are sliced in different ways.
3. Artistic designs could be created using geometrical instruments and also by means of curve stitching.

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Mathematics Enrichment, *Ripley and Tait*
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program four

GEOMETRY INVOLVED IN MODEL CONSTRUCTION

This program reinforces the idea that a knowledge of geometry is essential if Man is to continue to design and build even the most basic structures in this world.

PROGRAM CONTENT

Students gain a good deal of their early knowledge through experiences encountered during play. The program takes advantage of this aspect of student experience by having them illustrate methods of bricklaying with toy bricks as well as faulty and correct methods of tower building using model construction.

After moving to the classroom, students construct various polygons from wooden and plastic bars and illustrate that the triangle is the only rigid polygon. Three triangles with sides 6″, 7″, and 8″, are constructed and, by superimposing one on the other in turn, it is shown that they are identical in shape and thus we say they are congruent. However, when three quadrilaterals are constructed with sides 3″, 6″, 7″, and 9″, all three quadrilaterals are different in shape, because the sides are connected in different orders. The student audience is left with the question, "in how many different ways could these four sides be put together to form different quadrilaterals?" The answer to the question is "three". The necessity of the triangle for rigidity in three-dimensional skeletons is also illustrated as students make models using pipe cleaners and drinking straws.

A brief history of linear measure is discussed, and the following relationships are illustrated:

British System	*Metric System*
1 yard = 3 feet = 36 inches	10 millimetres (mm.) = 1 centimetre (cm.)
1 rod = 5½ yards	100 centimetres = 1 metre (m.)
1 mile = 1760 yards	1000 metres = 1 kilometre (km.)

It may be of interest to know that the yard is now officially defined in Canada, the United States and the United Kingdom as $\frac{9144}{10000}$ of the international metre.

The international metre is defined as "1,650,763.63 wavelengths in vacuum of radiation corresponding to the transition between the levels of $2P_{10}$ and $5d_5$ of the krypton 86 atom".

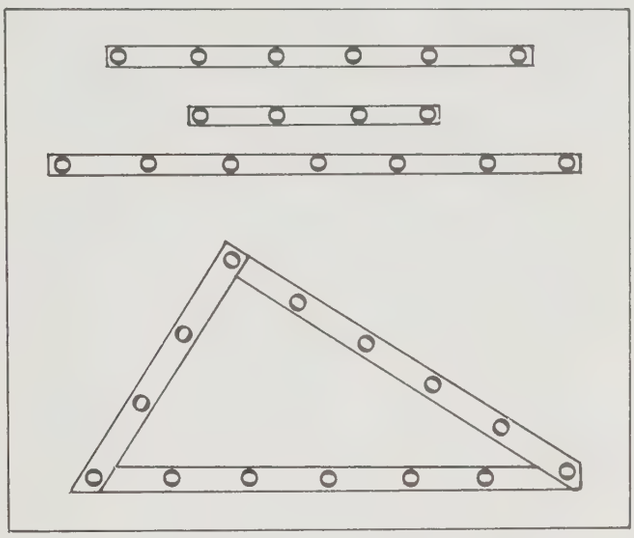
During the last portion of the program, the student audience is asked to participate. Instructions are given for the construction of triangles using ruler and compasses.

PREPARATION FOR TELECAST

Students should have pencils, rulers, and compasses on their desks so they can participate in the program.

The following materials would be useful in the follow-up activities after this program as well as for several other programs to come.

1. An assortment of flat bars of different lengths made from cardboard, wood, metal or plastic with holes drilled in them at regular intervals. Small bolts or paper fasteners are also necessary to make various polygons from these bars as shown.



2. A package of drinking straws and several dozen pipe cleaners.

SUGGESTED POST-VIEWING ACTIVITIES

1. Students could be encouraged to construct two- and three-dimensional models using the materials suggested. It is only through the construction of such models that students fully realize the importance of the triangle for rigidity.
2. As review of grade 7 work, students could be given practice constructing triangles with rulers, compasses and protractors. The following examples will be discussed in program 5, and it would be helpful to students if they had already done these and a few others like them before viewing.
   a. Construct triangle XYZ when XY = 7.2 cm., angle XYZ = 48° and YZ = 9.4 cm.
   b. Construct triangle DEF when angle DEF = 35°, EF = 10.4 cm., and angle DFE = 64°.
   c. Construct triangle MNO when MN = 3½ in., angle MNO = 30° and MN = 2¼ in.

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The Romance of Weights and Measures, *K. G. Irwin*
The Story of Mathematics, *Shaw and Fuge*
The Wonderful World of Mathematics, *Hogben*
The Grant Golden Book of Mathematics, *Alder*

program five

TO CONSTRUCT AN ANGLE EQUAL TO A GIVEN ANGLE

This program illustrates methods of angle construction in the real world and teaches students to construct angles equal to angles using only ruler and compasses.

PROGRAM CONTENT

The program opens with a short historical discussion of the origin of geometry. We are told that the word itself originated over 2,000 years ago in Egypt, and was derived from two Greek words meaning "earth" and "measure". Apparently farmers along the Nile River were forced to abandon their farms for a period of time each spring due to floods. After the floods had subsided and they returned, the land formation had changed shape, and they had to re-survey their farms. In these surveys, certain rules were established and these became known as the rules of Geometry.

The first Geometry was very practical, and was mainly concerned with construction and measurement. However, when Greek civilization spread to Egypt, many of the Greek mathematicians began to study why the methods worked. Thus they created and organized a more theoretical or deductive type of geometry.

The next portion of the program completes and discusses the constructions which were suggested by questions 2a, 2b and 2c in the follow-up activities of program 4. It is noted that a unique triangle will be determined under the following conditions:

1. If the lengths of the three sides are given
2. If the lengths of two sides and size of the contained angle are given
3. If the lengths of one side and the size of two angles are given

However, it is also pointed out that sometimes more than one correct triangle can be drawn from information which gives the lengths of two sides and an angle opposite one of them.

Students are then led to discover that (1) in an isosceles triangle, the angles opposite the equal sides are equal, (2) if one angle of a triangle is greater than another, the side opposite the greater angle is greater than the side opposite the lesser angle.

Several practical examples of angle construction as used by carpenters are shown before a ruler-and-compasses method of constructing an angle equal to a given angle is developed. The remainder of the program allows students to practice this basic construction.

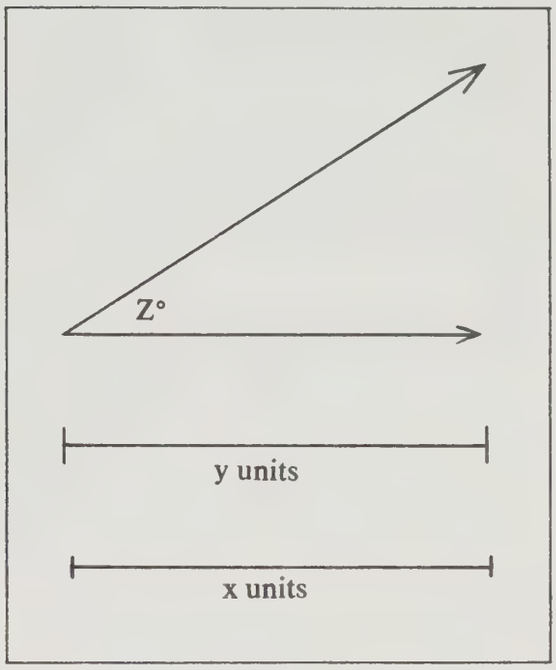
PREPARATION FOR TELECAST

Students should have their solutions to questions 2a, 2b, and 2c from the follow-up activities of program 4 available as these will be discussed in the early part of this program. Ruler, compasses and protractors are required for participation in the program.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss any part or parts of the program which created particular interest or difficulty. Students may be able to contribute other methods for angle constructions.

2. Students could be encouraged to bring practical construction tools and show them to the class.
3. Questions such as the following could be given to reinforce angle construction using ruler and compasses:
   a. Construct a triangle ABC if AB = x units, BC = y units and angle ABC = Z°.



   b. Have the students draw a line segment z and two angles X and Y. Then have them construct a triangle ABC so that BC = z, <ABC = X, and <ACB = Y.
   c. Construct any two angles and call them PQR and MNO. With ruler and compasses only, construct an angle equal to the sum of angles PQR and MNO.
   d. Construct any triangle ABC. With ruler and compasses only, construct an angle equal to the sum of angle A, angle B, and angle C.
   e. Have the students draw a line segment z units and two angles X° and Y°. Then have them construct a triangle ABC so that BC = z units, angle ABC = X°, angle ACB = Y°.

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program six

TO BISECT A LINE SEGMENT

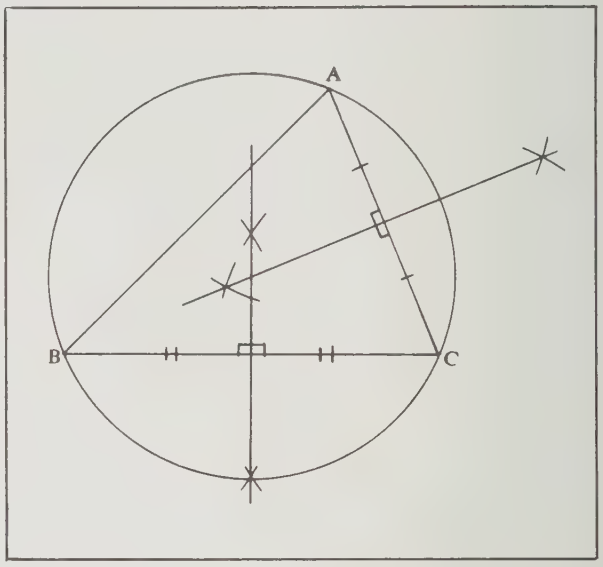
This program is designed to help students discover how to bisect a given line segment and to gain insight into the process.

PROGRAM CONTENT

Thales of Miletus (624-548 B.C.) is often called the Father of Geometry because he is the first known individual with whom mathematical discoveries are associated. Several amusing anecdotes about Thales are illustrated.

Students divide a rope into two, three, and four equal parts without using formal measurement. Then the student audience is asked to participate as a line on a page is bisected by folding. By examination of this method, students are led to discover a method of right bisecting a line segment using only ruler and compasses.

The audience participates in doing several examples which reinforce the idea of right bisection of a line segment, including an example where they construct the circum-circle of a triangle as shown below:



A short historical account of Euclid (about 365 B.C.) and his famous *Elements* is given. No work, except the Bible, has been more widely used or intensively studied. Over one thousand editions of Euclid's *Elements* have been printed since the invention of the printing press, and this work has dominated the teaching of geometry until recently. However, geometries other than "Euclidean Geometry" do exist, and one example involving the sum of the angles of a triangle on the surface of a sphere is illustrated.

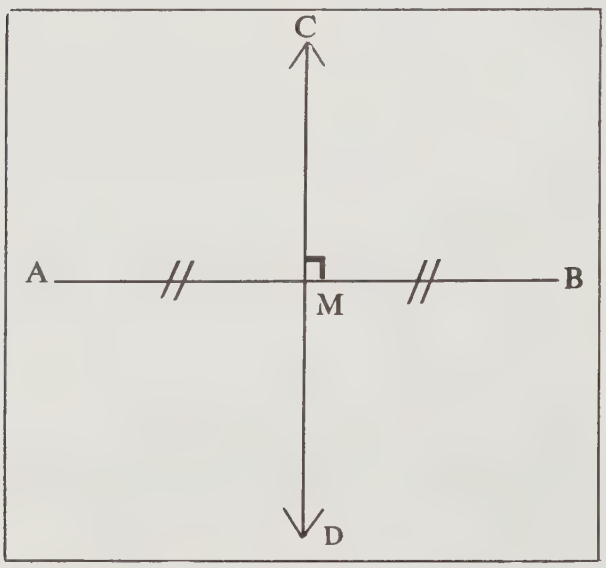
PREPARATION FOR TELECAST

To participate in the program students will need pencils, rulers, compasses and protractors. As well as their workbooks, they should have one or two sheets of *underlined* paper (tracing paper or onion skin), used for folding to match points and segments.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss the program regarding points of interest and difficulty. Note the difference between the "bisector" and "right bisector" of a line segment. The "right bisector" must bisect the segment at right angles.

Students should be encouraged to mark each diagram to indicate the construction.



For example, if CD is the right bisector of segment AB, then they should mark $AM = MB$ *and* $\angle CMB = 90°$.

2. Questions such as the following could be given to reinforce the methods and concepts involved in this fundamental construction, namely, to bisect a given line segment:

   a. Draw any line segment MN and divide it into four equal parts, using only ruler and compasses.

   b. Draw a segment XY and then right bisect it. Mark points A, B, C, and D on the right bisector. With centres A, B, C, and D and radii AX, BX, CX, and DX, draw four circles. Do they each pass through Y? If so, why?

   c. Construct the triangle ABC so that AB = 3 in., BC = 4 in., and AC = 2¾ in. Construct right bisectors to cut AB at X and AC at Y. Join XY and measure its length. What relationships do you see between XY and BC? (XY is parallel to BC and equal to one-half of it.)

   d. Draw a circle with centre O and radius 2½ in. Draw any chord AB in the circle and then right bisect this chord. Does this right bisector pass through any significant point? If a circle is given, could you find its centre by expanding the above method? (If two non-parallel chords are drawn, the right bisectors will intersect at the centre.)

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An Introduction to the History of Mathematics, *Eves*
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program seven

TO BISECT AN ANGLE — To construct a line perpendicular to a line from a point in the line

This program is designed to show students some practical methods and needs for bisecting angles as well as to help them discover how to bisect an angle using only a ruler and compasses. The construction of a perpendicular to a line from a point in the line is introduced through the bisection of a straight angle.

PROGRAM CONTENT

In earlier programs, we noticed that if polygons were made from plastic bars of specific lengths, the triangle was the only one that formed a rigid figure. The quadrilateral was treated lightly because of its flexibility. However, this program illustrates that many relationships remain the same in the quadrilateral, even though the shape changes. These relationships are analyzed and used to develop a method of bisecting an angle using only ruler and compasses.

Other methods of bisecting an angle are illustrated through paper folding, the use of a board, and the carpenter's square. Several examples are given which show a practical use for angle bisection.

The student audience is asked to participate in doing several examples involving the bisection of angles using only rulers and compasses. In one case, they bisect a straight angle and obtain two right angles. Further bisections allow them to construct angles of 45 degrees and 22½ degrees.

While it is extremely easy to bisect any given angle using only straight edge and compasses, it is impossible to trisect such an angle with these instruments. However, many devices have been devised that will trisect angles. The "tomahawk", which is shown in figure 1, is one of them.

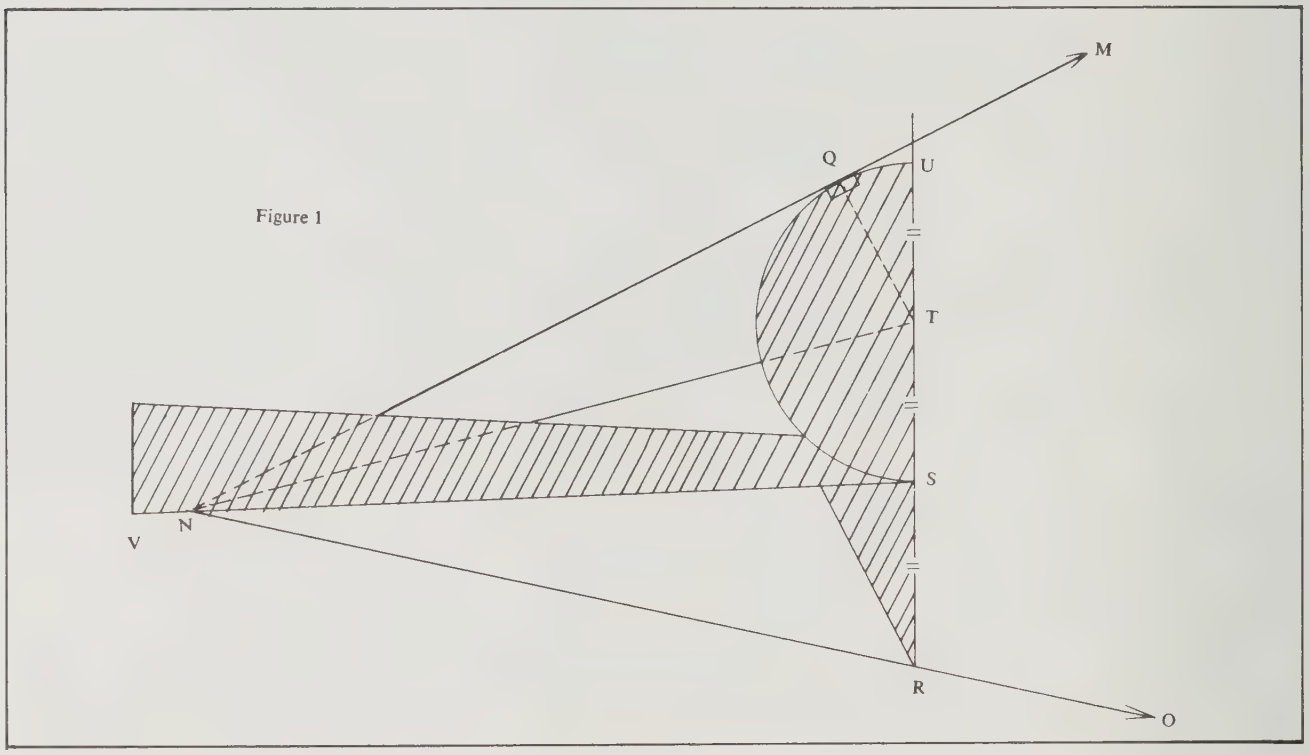

Figure 1

Because the "tomahawk" can be constructed using only straight edge and compasses, the student audience is asked to construct one.

In figure 1, the shaded part represents a "tomahawk" which has been cut out of cardboard. It has been applied to angle MNO so that the angle is trisected by NS and NT. Note triangles RNS, SNT and TNQ. The program ends as students demonstrate the use of the "tomahawk" in trisecting angles.

PREPARATION FOR TELECAST

The usual geometric instruments are needed as the student audience is asked to participate in a good deal of the program. An assortment of the plastic, wood, metal, or cardboard bars as required in program 4 is used during the program and would be useful in the follow-up activities.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest and difficulty in the program and encourage students to look for instances where angle bisectors are used.
2. Students could be given the opportunity to devise and use simple tools, like those mentioned in the program, to bisect and trisect angles.

3. Questions such as the following could be assigned to reinforce methods and concepts in the two fundamental constructions which were discussed:

   a. On the same diagram and using only ruler and compasses, construct and mark angles of 90 degrees, 45 degrees, 22½ degrees, 67½ degrees, 112½ degrees and 11¼ degrees.

   b. Draw any acute angle ABC and then construct BD to bisect angle ABC. Let P and Q by any points on BD. Using a set square construct PR and QS perpendicular to BC and meeting BC at R and S respectively. Draw circles with centres P and Q and radii PR and QS. What do you notice about these circles?

   c. Construct two lines AB and CD to intersect at E. Construct EF and EG to bisect angles AED and AEC respectively. Measure angle FEG.

   d. The mariner's compass has 32 directions. To draw such a compass construct a large circle centred on a page. Then construct a diameter to mark the East-West directions. Another diameter can be drawn perpendicular to this which shows the North-South directions. By continually bisecting angles at the centre, 16 diameters can be drawn and thus the 32 equally spaced points of the compass will be indicated on the circle. Nine of these directions in order from North to East have the following names and abbreviations: North (N), North by East (N/E), North-Northeast (NNE), Northeast by North (NE/E), East-Northeast (ENE), East by North (E/N), East (E). Construct the compass and by extending the pattern indicated by these names, label the 32 directions on it.

Bibliography

An Introduction to the History of Mathematics, *Eves*, Holt Rinehart

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program eight

TO DRAW A LINE PERPENDICULAR TO A LINE FROM A POINT OUTSIDE THE LINE

This program leads students toward the discovery of how to draw a line perpendicular to a line from a point outside the line, by reviewing and analyzing the former three fundamental constructions. Some work on parallel lines is also introduced so that students can discover relationships between alternate and corresponding angles.

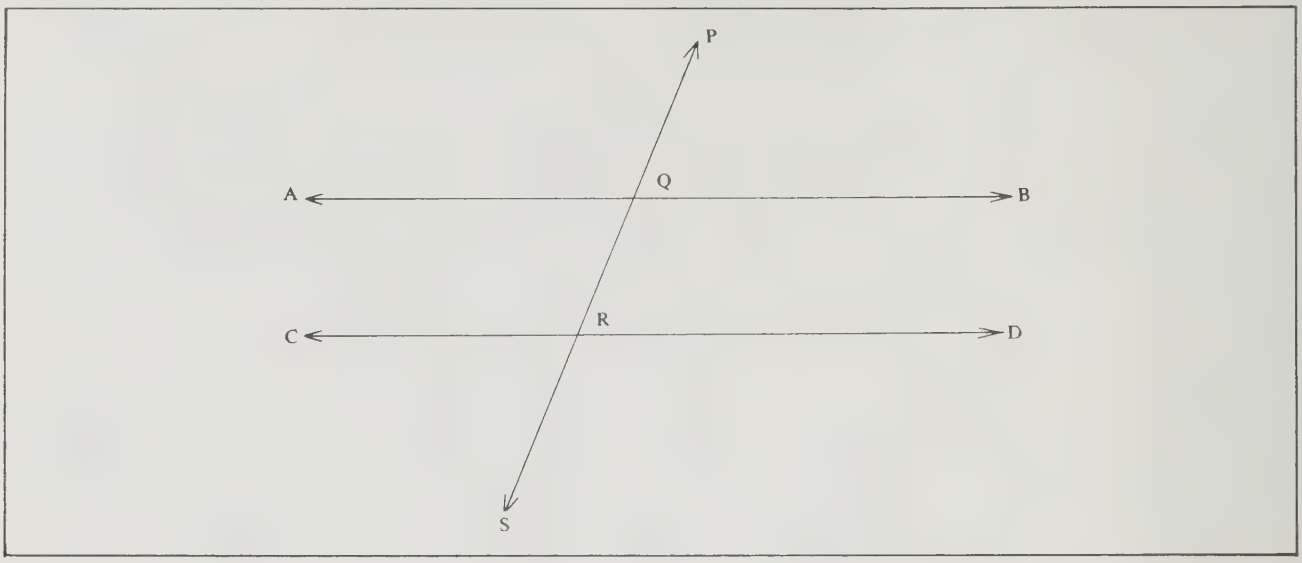
PROGRAM CONTENT

In the opening portion of the program, the student audience is asked to participate in the review of the following fundamental constructions:

1. To right bisect a line segment
2. To bisect an angle
3. To construct a line perpendicular to a line at a given point on the line

After analysing the concepts involved in these constructions, the student audience participates in the development of a method to construct a line perpendicular to a line from a point outside the line. This is followed by several examples which reinforce the method.

The next portion of the program is devoted to the study of parallel lines. When a line PS cuts two lines AB and CD, as shown in the diagram below, it is called a "transversal".



Pairs of angles such as AQP and CRQ or BQR and DRS are called "corresponding angles", whereas angles such as AQR and QRD or BQR and QRC are called "alternate angles". If the lines AB and CD are parallel, students discover through measurement that the angles in each pair of alternate or the corresponding angles are equal.

The program closes with an illustration showing how Eratosthenes (274-194 B.C.) measured the circumference of the earth. One of the key points in his solution applied the idea that "if a transversal meets two parallel lines, the corresponding angles are equal".

PREPARATION FOR TELECAST

In order to participate in the program, the students will need rulers, compasses and protractors along with their workbooks.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss the program and any difficulties arising from the fundamental constructions taken so far. The students might experiment with constructing the three altitudes of a given triangle (acute, obtuse) and thus promote a discussion of concurrent lines.
2. Corresponding angles and alternate angles could be reviewed so that each student is fully aware of their positions with respect to parallel lines and a transversal.
3. The story of how Eratosthenes measured the earth's circumference is well known but could lead to an interesting discussion. This is a case in which simple mathematics but great insight and understanding produced an amazing result.
4. Questions such as the following may be assigned to give further practice in the fundamental construction:
   a. Construct a triangle ABC such that AB = 2½ in., BC = 3½ in., and AC = 2¾ in. Construct a line from A perpendicular to BC, meeting BC at D. AD is

said to be an altitude of the triangle. Construct the other two altitudes from B and C. What do you notice about these three altitudes? (An altitude of a triangle is a line drawn from one vertex perpendicular to the opposite side.)

b. Construct triangle PQR such that PQ = PR = 10.4 cm. and QR = 8.0 cm. Draw a line from P perpendicular to QR meeting QR at S. Measure QS and RS.

c. Construct a triangle ABC such that AB = 12 cm., BC = 14 cm. and AC = 9 cm. Bisect angles A and C and let the bisectors meet at O. Construct OD perpendicular to BC meeting BC at D. With centre O and radius OD draw a circle. (This is called the inscribed circle of a triangle.)

Bibliography

The Story of Mathematics, *Shaw and Fuge*
Mathematics Enrichment, *Ripley and Tait*
The Wonderful World of Mathematics, *Hogben*

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program nine

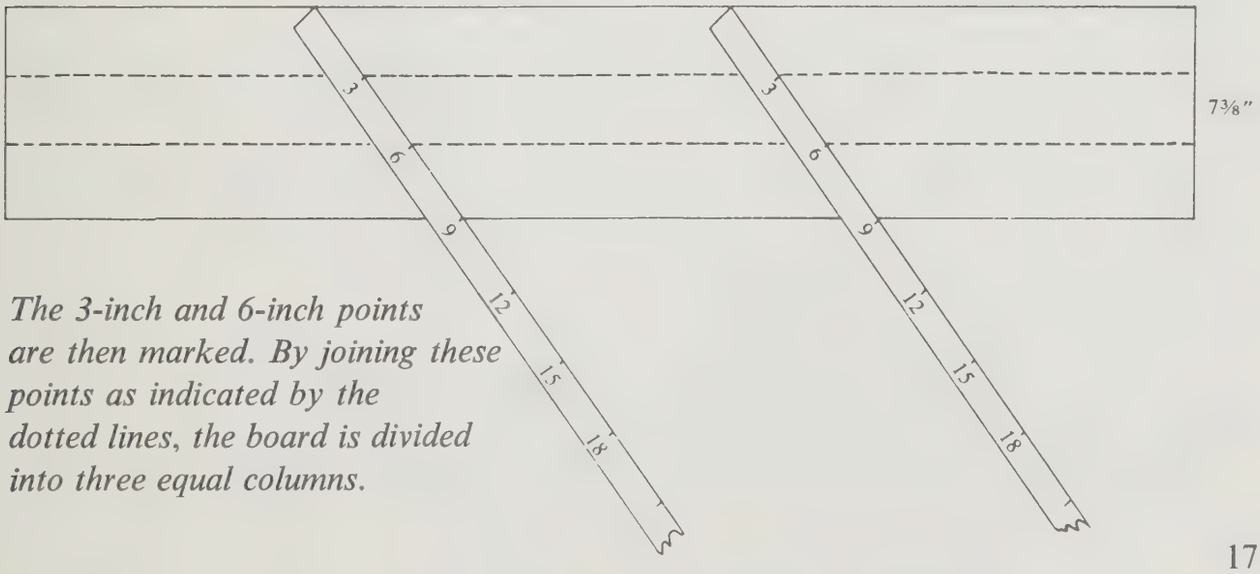
TO CONSTRUCT A LINE PARALLEL TO A GIVEN LINE

This program is designed to help students realize the usefulness of parallel lines and to teach them to construct a line parallel to a given line through a given point outside the line.

PROGRAM CONTENT

The program opens showing students manipulating quadrilaterals made from plastic bars. It is noted that if the opposite sides of a quadrilateral are made equal, the sides remain parallel, no matter how they are flexed. By analyzing such a model, students are led to discover how to draw a line perpendicular to a given line through a given point outside the line when using only ruler and compasses.

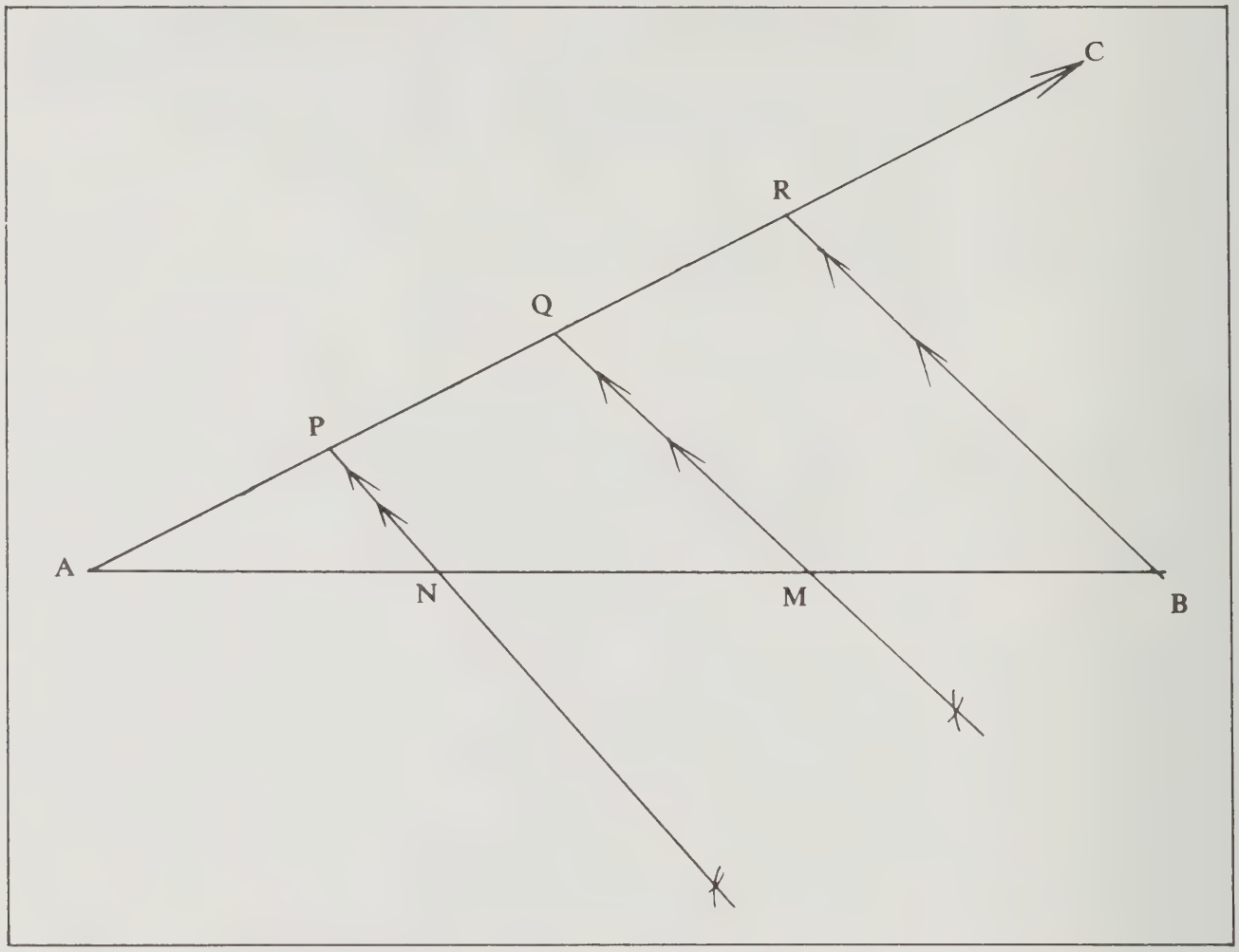
Several practical examples of the use of parallel lines and parallelograms are discussed. The teacher could ask the students to point out parallel lines in the various pictures shown. One of these shows the way a carpenter with a knowledge of geometry could divide a board $7\frac{3}{8}''$ wide into three equal parts with a minimum of calculation. A ruler can be placed in each of the positions as shown in the figure below:



The 3-inch and 6-inch points are then marked. By joining these points as indicated by the dotted lines, the board is divided into three equal columns.

The audience participates by doing a similar example where a page is divided into three equal columns. For this the students might use notebook paper which is 8½ inches wide by 11 inches long, and divide the *width* into 3 sections.

The program closes with an example illustrating how a line segment AB can be divided into three equal parts using only straight edge and compasses.



A ray, AC, is drawn and then equal segments AP, PQ and QR are marked on it, as shown. Then RB is drawn and QM and PN are drawn parallel to RB. By measurement, it can be shown that $AN = NM = MB$.

PREPARATION FOR TELECAST

To participate in the program, students need the usual geometric instruments including ruler, compasses, and protractor. An assortment of plastic, wooden or paper bars as described in the program 4 notes would be useful when discussing the program during the postviewing activities. This particular program permits oral participation by the students.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss the different methods used for constructing parallel lines. Bring a spirit level to class and discuss the bubble positions for parallel lines.
2. Students could construct models involving parallelograms from the plastic, wooden or paper bars and then observe relationships as they are flexed.

3. Examples such as the following could give practice in the ruler-and-compasses method of constructing parallel lines.
   a. Draw any triangle ABC and then construct a line through A parallel to BC. Find alternate angles in the diagram and measure them.
   b. Construct a triangle ABC such that AB = 3 inches, AC = 3½ inches, and BC = 4 inches. Mark D as the mid point of AB. Through D, draw a line parallel to BC meeting AC at E. Measure AE, EC, DE and BC. Note any relationships which appear significant. (AE = EC and BE = ½BC).
   c. Construct a line segment YZ, about 6 inches long. Then draw a ray YX such that angle XYZ is an acute angle. With suitable radius, cut off five equal segments YA, AB, BC, CD, and DE on the ray YX. Join EZ. Now draw lines through A, B, C, and D parallel to EZ and meeting YZ at F, G, H, and I respectively. Compare the lengths of the segments YF, FG, GH, HI, and IZ.

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program ten

PERIMETER AND CIRCUMFERENCE

This program is designed to reinforce the concepts of perimeter and circumference. Oral participation is permitted early in the program.

PROGRAM CONTENT

The program begins with a review of closed curves and then introduces the concept of perimeter as the *length* of the curve. If the closed curve is a polygon, the perimeter is found by taking the sum of the lengths of the sides. Shorter methods are indicated for special polygons, but no formulas are given. Measuring the perimeter or circumference of a circle is somewhat more difficult, but students achieve this in different fashions.

Two experiments are shown which develop the relationship $C = \pi d$ where "C" and "d" represent the measures of the circumference and diameter of a circle. Different values for π are mentioned, and a short historical account of how Archimedes (287-212 B.C.) calculated a value for it is illustrated and described. It is noted that modern computers have calculated π to several thousand decimal places, but in the problems discussed the *approximate* values of 3.14 and 22/7 are used.

(You may be interested to know that $\pi = 3.14159265358979323846$ when it is calculated to 20 decimal places.) The program closes after the student audience has participated in the calculation of the perimeters of several figures.

SUGGESTED POST-VIEWING ACTIVITIES

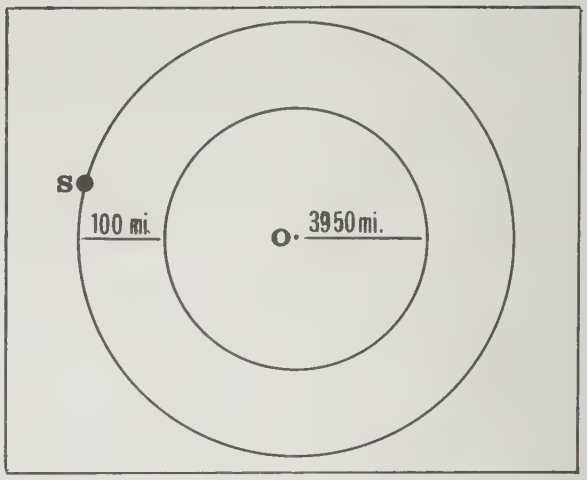
1. Discuss the concept of perimeter and have the students give practical examples that would require the calculation of perimeter—e.g. the length of wallpaper border required for a room.
2. The class could find its own value for π through experiments similar to those used in the program, and the results could be shown in graphical form.
3. Students could be encouraged to find the actual perimeter of rooms, window frames, hallways, and the school yard by measurement and calculation.

4. The solution to the satellite problem from the telecast is as follows:

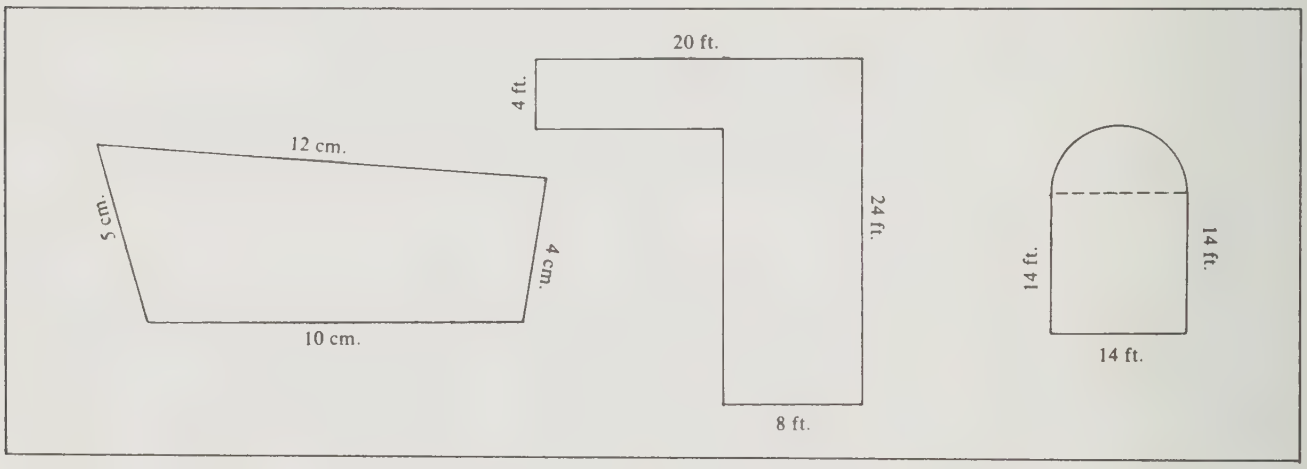
$$r = 3950 + 100$$
$$= 4050$$
$$\pi \doteq \frac{22}{7}$$

$\therefore$ the number of miles the satellite would travel in orbiting the earth is: $7C = 7 \times 2\pi r$

$$= 7 \times 2 \times \frac{22}{7} \times 4050$$
$$= 178{,}200$$



5. The Archimedes "Crown Problem" mentioned in the program could be further investigated in the reference book: MATHEMATICS, The Life Science Library.
6. Additional practice could be given by asking students to find the perimeters of figures similar to the following:



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program eleven

REGIONS AND AREA

The aim of this program is to give students a sound understanding of the concept of area.

PROGRAM CONTENT

In order to establish the concept of the size of a region, students are asked to estimate how many sheets of construction paper it will take to cover a bulletin board. Each piece of paper is considered an arbitrary unit of size, and since 7½ sheets of paper are required, the area of the bulletin board is said to be 7½ units.

The next example shows the same bulletin board covered with 18 unit triangles. Thus the area is said to be 18 units, as each unit is the size of one of these triangular pieces of construction paper. Either the triangular or rectangular regions are shown to be suitable for measuring area, but some time is spent experimenting to see if circles, octagons, hexagons or pentagons would also be suitable units.

After the concept of area is introduced, an experiment is carried out to show relationships between perimeter and area in equilateral triangles. Then examples are shown to indicate that a closed curve with a constant perimeter does not necessarily have a constant area and vice versa.

The square inch and the square centimeter are defined and then areas of irregular figures are determined by tracing their shapes on squared paper and counting the number of units they cover.

PREPARATION FOR TELECAST

In order to participate in the program the students need the usual geometric instruments and a workbook, with a table similar to figure 1:

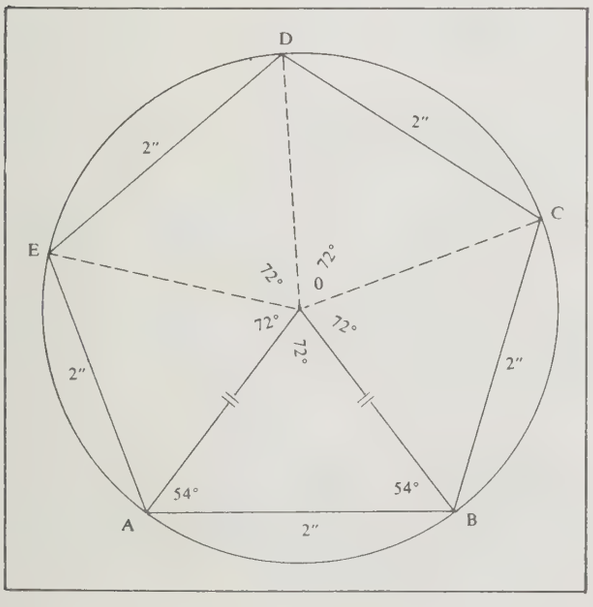
Relationships Between Perimeter and Area in Equilateral Triangles		
Measure of length of each side	*Measure of perimeter*	*Measure of Area*

For the follow-up activities, it would be convenient to have a supply of:

—graph paper with some marked in square inches and some in square centimetres

—identically pre-cut circles, triangles, squares, parallelograms, regular pentagons, regular hexagons, and regular octagons. These are to be fitted together to form patterns, the sides of each polygon should be a constant length, e.g. 2 inches.

The construction of a pentagon may give some difficulty, but it (or any other) can be constructed by using the following idea:



Each angle at the centre of the circle was calculated to be $360° \div 5 = 72°$. *Then each base angle of the isosceles triangles was calculated to be* $(180° - 72°) \div 2 = 54°$. *In the actual construction, begin by drawing isosceles triangle AOB. Then with centre O and radius OA, draw a circle. Points C, D and E may be found by constructing 72° angles at the centre or cutting off arcs equal to arc AB on the circumference.*

SUGGESTED POST-VIEWING ACTIVITIES

The program could be discussed and students encouraged to carry out activities which would reinforce the concept of area. For example:

a. Compare the areas of different surfaces by covering them with identical pre-cut polygons.

b. By using graph paper, determine the area of irregular figures.

c. Compare the size of countries or lakes on a map by counting the number of equal square regions that each covers.

Bibliography

Mathematics for Primary Schools, Bulletin No. 1, *British Schools Council*

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program twelve

UNITS FOR MEASUREMENT OF AREA

This program attempts to give students an understanding of the different units which are commonly used to measure area and of some of the relationships between them.

PROGRAM CONTENT

The formula ($A = l \times w$) for finding the area of a rectangle is developed by observing the areas of rectangular regions marked on graph paper. Several units of square measure are introduced from both the British and the Metric systems and the following relationships are established:

British System	*Metric System*
1 sq. ft. = 144 sq. in.	1 sq. cm. = 100 sq. mm.
1 sq. yd. = 9 sq. ft.	1 sq. m. = 10,000 sq. cm.
1 sq. rd. = 30¼ sq. yd.	1 hectare = 10,000 sq. m.
1 acre = 160 sq. rd.	
1 sq. mi. = 640 acres	

Several problems are assigned which involve conversions between units within each system of measure.

Two methods of land survey are discussed briefly. In the new survey as used in western Canada and northern Ontario, farms are usually 160 acres and there are four of them in a square mile. In many older surveys, concession roads are 1¼ miles apart, dividing the land into blocks containing 1,000 acres. These, in turn, are subdivided into ten farms of 100 acres each.

The program closes after the student audience has had an opportunity to calculate the area of several figures and to view sample solutions.

PREPARATION FOR TELECAST

In order to participate in the program, the students need the usual geometric instruments and workbooks.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss the topics of interest and clarify points of difficulty.
2. Students should be encouraged to make charts indicating the relationships between units. For example, a square with each side one foot long may be divided into 144 squares with each side one inch long. A scale diagram could indicate that one square rod is the same size as 30¼ square yards.
3. Students could be encouraged to find out how the land in their vicinity was surveyed. Many cities and towns were originally surveyed as townships and the main streets were formerly concessions or crossroads.
4. Students could be given the opportunity to find the area of convenient surfaces around the school, through measurement and calculation, e.g. the area of the classroom floor, the area of blackboard surface, the area of window surface, the area of the outside walls of the school, the area of the school yard.
5. Questions such as the following could be assigned to give additional practice in calculating area:

   a. Find the area of the figure below and express the answer in square centimetres.

   b. If a rectangular driveway is 15 feet wide and 63 feet long, find its area in square yards. Find the total cost to pave the driveway at the rate of $4.50 per square yard.

   c. If a rectangular field is 40 rods by 24 rods, find its area in acres.

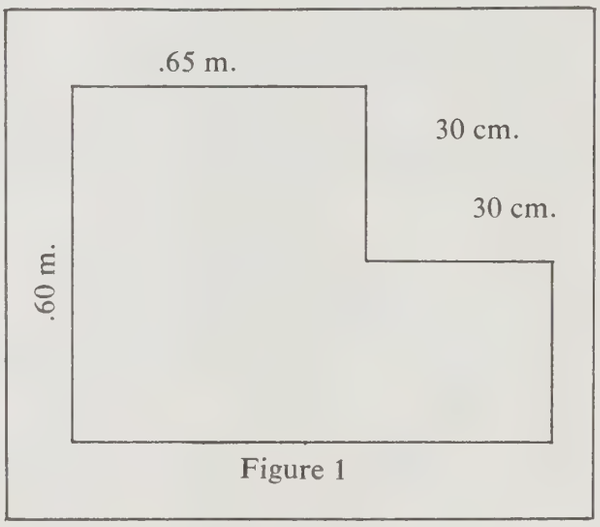

Figure 1

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program thirteen

DEVELOPING FORMULA FOR CALCULATING AREA

This program is designed to help students understand and develop formulas for finding the area of parallelograms, triangles and circles.

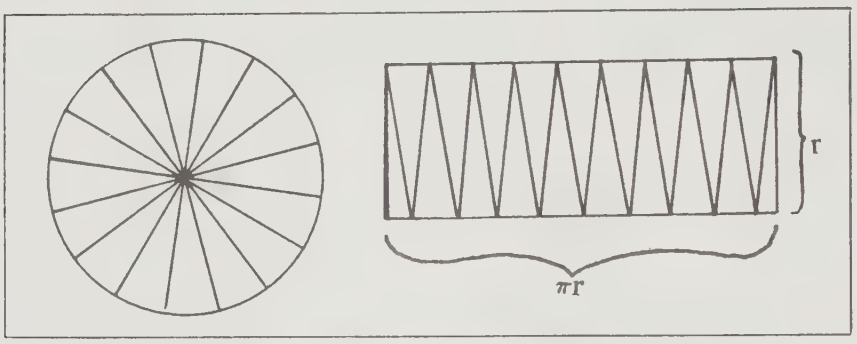
PROGRAM CONTENT

The program begins by reviewing the formula for the area of the rectangle: A = lw. The parallelogram is then considered, and it is cut and rearranged to form a rectangle of equal area. Then the formula for the area of a parallelogram is established as

A = bh where "b" represents the length of one side of the parallelogram, and "h" represents the perpendicular distance from it to the opposite side.

A diagonal is drawn in a parallelogram and we see that this divides it into two congruent triangles. Thus, the area of each triangle is half the area of the parallelogram. It is then shown that any two congruent triangles can form a parallelogram. Thus, the formula $A = \frac{1}{2}bh$ is developed for finding the area of a triangle where "b" represents the length of one side and "h", the length of the corresponding altitude.

The formula $A = \pi r^2$ is developed for finding the area of the circle. A model with the circle divided into about 50 sectors is decomposed and rearranged to form a rectangle with length "πr" and width "r". A simplified version of this is shown in the diagrams:



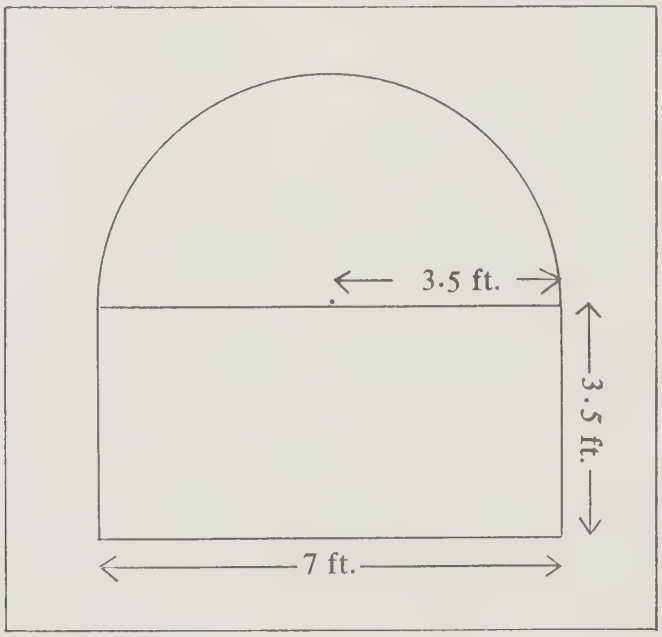
Several examples are given to reinforce the concepts and methods involved in calculating area using formulas.

PREPARATION FOR TELECAST

Students need the usual geometric instruments and workbooks to participate in the program. Construction paper or Bristol board would be useful in the follow-up activities.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss the program and elaborate on points of interest and clarify points of difficulty.
2. The solution to the "Archway" problem from the telecast is as follows:



a. $d = 7$, $r = 3.5$, $\pi \doteq \frac{22}{7}$ ∴ the area of the semi-circle in square feet is given by:

$$A_1 = \tfrac{1}{2}\pi r^2$$
$$= \frac{1}{2} \times \frac{22}{7} \times 3.5 \times 3.5$$
$$= 19.25$$

b. $l = 7$, $w = 3.5$ ∴ the area in square feet of the rectangular region is given by:

$$A_2 = lw$$
$$= 7 \times 3.5$$
$$= 24.5$$

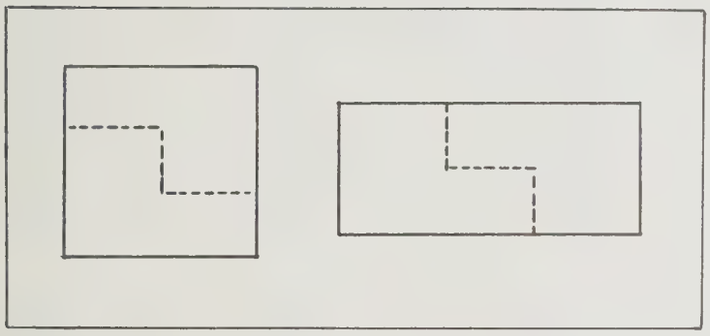
c. ∴ the area in square feet of the archway is obtained by:

$A = A_1 + A_2$
$= 19.25 + 24.5$
$= 43.75$
$= 44$ (nearest square foot)

3. Students could be encouraged to cut and rearrange figures in different ways, for example:

   a. Cut and rearrange a triangle to form a parallelogram.

   b. Cut and rearrange a rectangle to form a square. Can this be done with one cut? (see figure below)

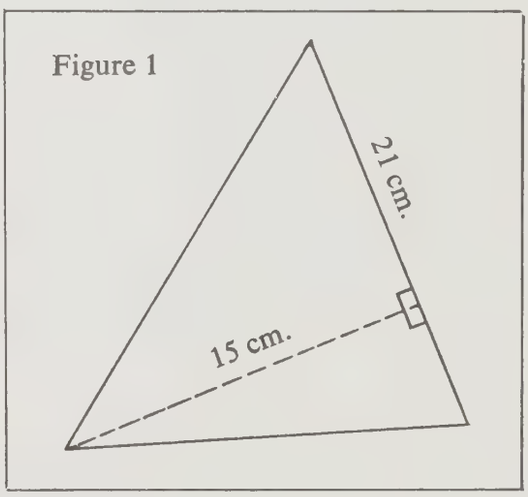


Examples such as these will give insight into concepts behind formulas and will also reinforce the idea of conservation of area.

4. Questions such as the following could be given to allow practice in using the formulas:

   a. Find the area of the triangle in figure 1.

   b. By using formulas already developed, find the area of the trapezoid in figure 2.



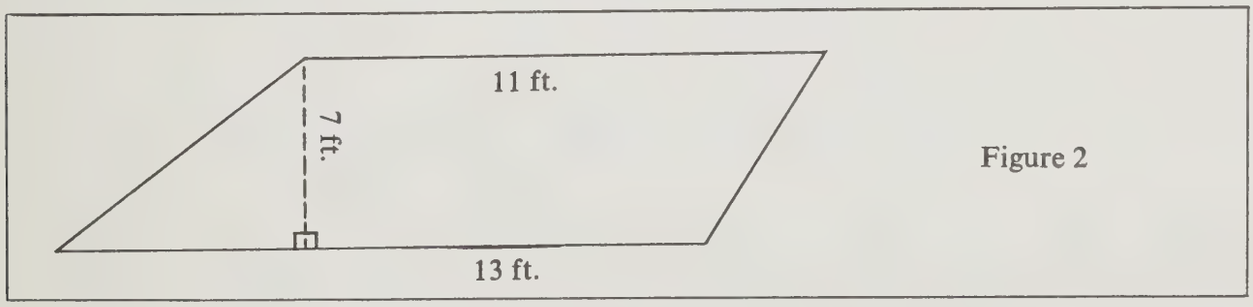
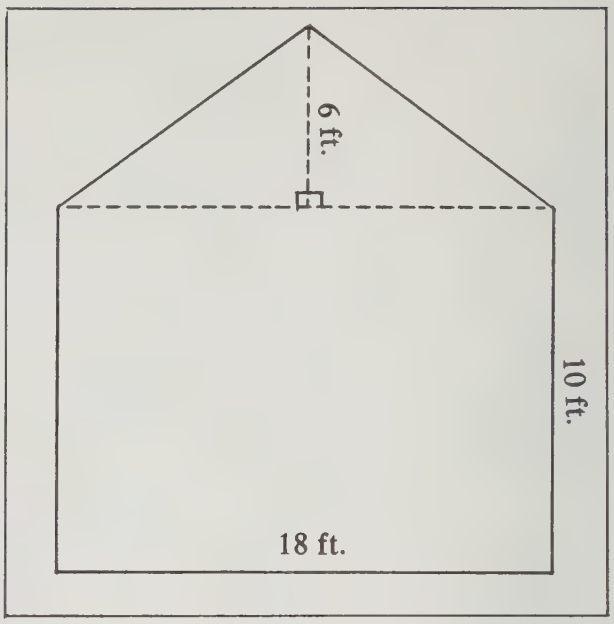


Figure 2

c. Calculate the area of the end of a house with dimensions as shown.



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program fourteen

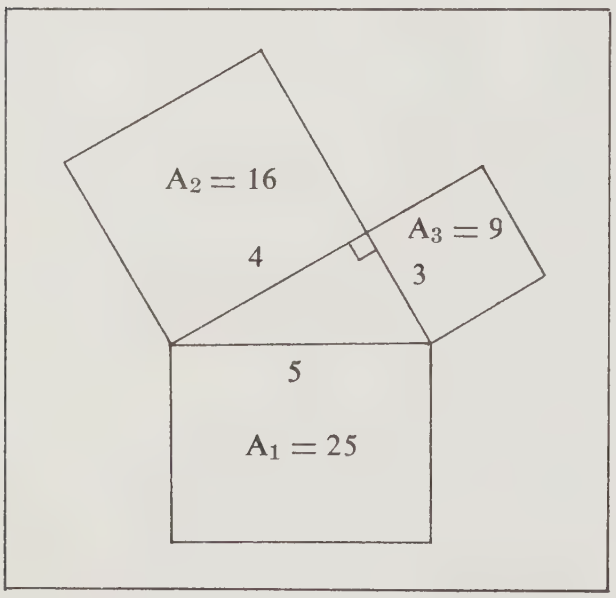
THE PYTHAGOREAN THEOREM

This program introduces students to the Pythagorean theorem and to some simple applications of its results.

PROGRAM CONTENT

The program begins as students try to arrange the plastic bars which were used in previous programs, to form right-angled triangles. They find this is possible if the sides are 3 inches, 4 inches, and 5 inches; 6 inches, 8 inches, and 10 inches; 9 inches, 12 inches, and 15 inches; and 5 inches, 12 inches, and 13 inches. They note the pattern of the first three sets of numbers, but at this time they do not see any relationship between the numbers in any one set.

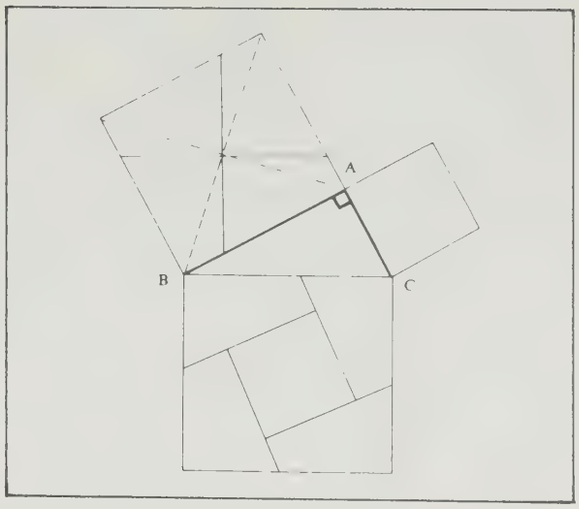
Squares are then drawn on the sides of some of the right-angled triangles, as shown below:



It is then shown in each case that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

The following example illustrates the Pythagorean theorem another way:

ABC is any right-angled triangle, and squares made from cardboard are placed on each side as shown. The centre of the square on AB is located by joining the diagonals as indicated by the dotted lines. The other two lines through this centre point are drawn parallel and perpendicular to the hypotenuse BC. When the square on AB is cut along these lines, the pieces are rearranged so that the squares on AB and on AC exactly cover the square on BC as shown.



A short historical account of Pythagoras is then given. Pythagoras was born about 572 B.C. on the island of Samos off the coast of Turkey. He founded the Pythagorean Order where the members banded together in a closely knit society with secret rites and observances. They studied philosophy, mathematics and natural science. Their geometry was closely related to arithmetic, and through such a study they actually began the branch of mathematics called "Number Theory". Due to political pressures the order was forced to separate and Pythagoras died, or was murdered, about 507 B.C. While the Pythagorean theorem is credited to him, his actual method of proof is not known.

Based on the Pythagoras theorem, several questions are assigned and solutions discussed. At the end of the program, the following problem is assigned: a vertical cliff 39 feet high is adjacent to a river 52 feet wide. What length of rope is required to connect the top of the cliff to the opposite side of the river?

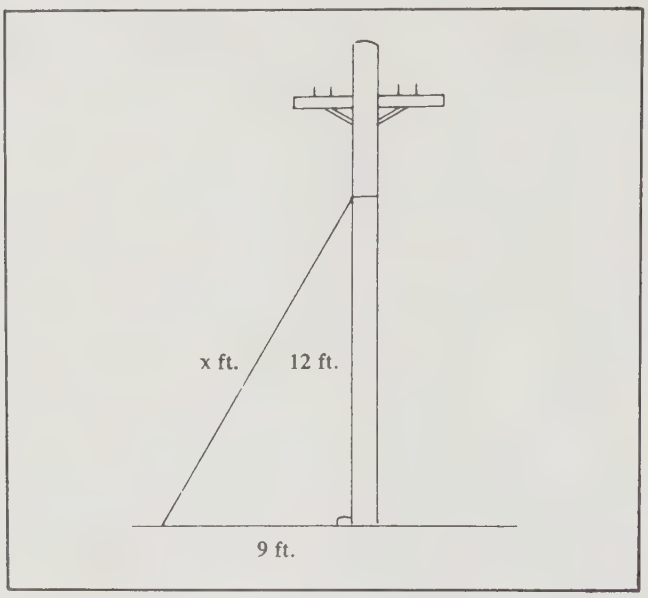
PREPARATION FOR TELECAST

The usual geometric instruments and workbooks are needed for student participation in the program. It would be an advantage if the students had covered the work on square root (curriculum 1.12B, unit 3:7) before watching this program. Construction paper or Bristol board would be useful for post-viewing activities.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.
2. Students could be encouraged to illustrate the Pythagorean theorem for any right-angled triangle by other paper-cutting methods.
3. Whole numbers which can represent the lengths of the sides of a right-angled triangle are called "Pythagorean Triples" (e.g. 3, 4 and 5 form a Pythagorean Triple since $3^2 + 4^2 = 5^2$). Students could be asked to find as many of these "triples" as they can.
4. Questions such as the following could be assigned to give the student practice using the results of the theorem:

If a telephone pole is to be braced by a guy-wire as shown in the diagram, find the length of the guy-wire.



5. The Pythagorean theorem for triangle ABC where angle ACB is a right angle is algebraically written as $c^2 = a^2 + b^2$. In view of this relationship $c^2 = a^2 + b^2$ for the right-angled triangle, the following question could be considered using the right-angled triangles in the program: What relationship, $c^2 > a^2 + b^2$ or $c^2 < a^2 + b^2$, is satisfied when angle ACB is less than 90°? is more than 90°?

6. During the program, the topic of number theory as introduced by Pythagoras is briefly mentioned. The geometric reasons for naming certain types of number such as triangular 1, 3, 6 . . . ; square 1, 4, 9, 16 . . . ; pentagonal 1, 5, 12, 22 . . . ; could be investigated. Students could be encouraged to discover relationships between the different types of numbers. Such relationships as "any square number is the sum of two successive triangular numbers" could be examined.

Bibliography

An Introduction to the History of Mathematics, *Howard Eves*

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program fifteen

SURFACE AREA OF THREE-DIMENSIONAL FIGURES

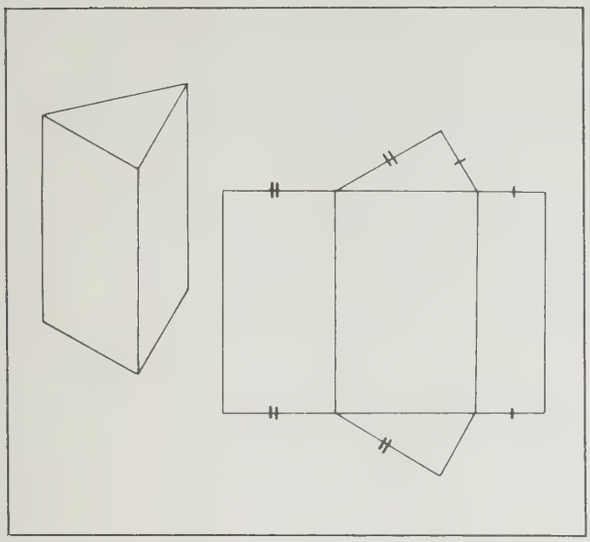
This program illustrates the finding of surface area by developing methods which the pupils will understand.

PROGRAM CONTENT

The program begins by reviewing the fact that some polygons fit together to cover a region and some do not. Then students illustrate that some polygons can be fitted together to form polyhedrons. Our attention is then turned to the surface area of polyhedrons and we begin by analyzing and calculating the total surface area of a cube.

The next geometric form to be dealt with is the "rectangular" prism or "cuboid". It is unfolded in two slightly different ways and the area calculated for each.

The area of the surface of the triangular prism is found by unfolding it as shown below and then the total area is determined as the sum of the areas of the two triangular bases and of the large rectangle which forms the lateral surfaces. Note that the length of this large rectangle is the same as the perimeter of the triangular base.



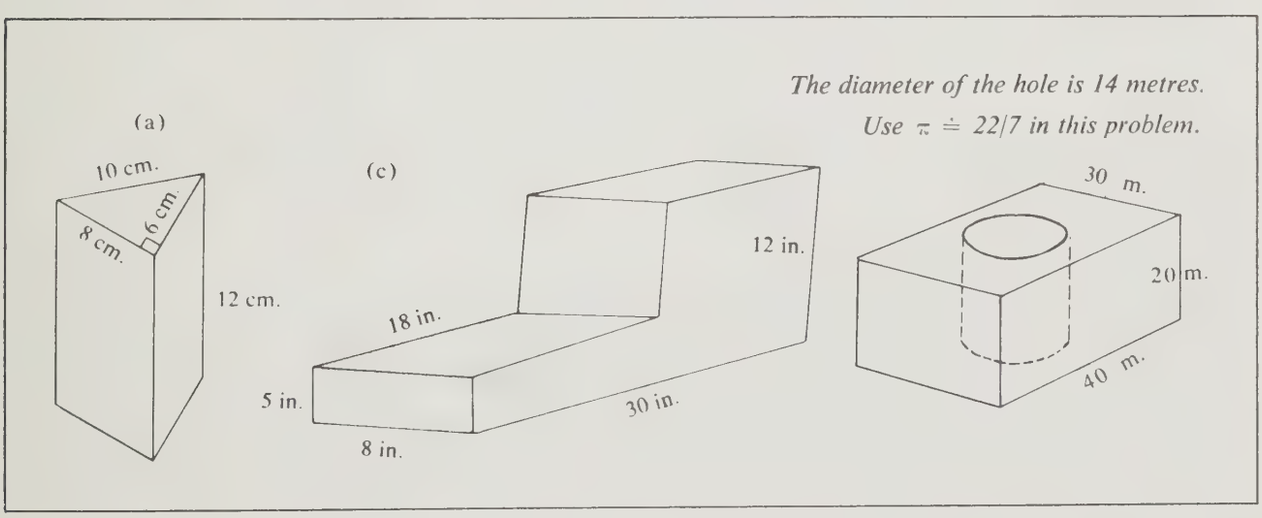
The last figure to be considered is the cylinder. It is shown that the total surface area is found by taking the sum of the areas of the circular bases and the large rectangle which forms its lateral surface. Again it is observed that the length of this rectangle is the perimeter of the base, which in this case is the circumference of the circle. Throughout the program students are asked to participate in calculating areas of different figures. The program ends with a problem left to be finished during class time.

PREPARATION FOR TELECAST

The usual geometric instruments and workbooks are needed if the student is to participate fully in the program. It is also essential that students have a good knowledge of area and perimeter in plain figures before they attempt to find the area of surfaces of three-dimensional figures. Precut polygons, construction paper, scotch tape and glue would be useful for post-viewing activities.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest and clarify difficulties which have arisen during the program. It is most helpful during such discussion if students can see and handle solid models.
2. Construction materials could be made available so that students could design and make different models. The regular polyhedrons could be made by taping precut polygons together, or by leaving tabs on the polygons and then gluing them together with the tabs inside.
3. Students could be asked to make a cone from construction paper (note that the lateral surface of a cone unfolds to form a sector of a circle.)
4. For additional practice, students could be asked to find the surface area of figures such as the following:



5. Questions such as the following could be assigned to give the students practice in calculating area: A gasoline drum in the shape of a cylinder is to be painted. Find the total area to be painted if the drum has a height of 20 feet and a circumference of 20 feet. Use $\pi \doteq \frac{22}{7}$

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program sixteen

INTRODUCTION TO VOLUME

This program is designed to give meaning to the concept of volume, and to introduce methods of calculating it.

PROGRAM CONTENT

The program opens as students compare the size of a pop bottle and a pop can by finding out which will hold the most water. Then they compare the volume of two rocks by finding which displaces the most water. In order to find out how much larger a cubical container is than a water pitcher, a "glassful" is defined as an arbitrary unit of volume. Then the volume of each container is measured in terms of this unit.

After discussing the need for standard units, the cubic inch is defined as the space occupied by a cube with each edge one inch in length. Similarly the cubic centimetre is defined as the space occupied by a cube with each edge one centimetre in length.

It is pointed out in the program that an object may have a volume of one cubic inch (or centimetre) without being in the shape of a cube.

Then other units are introduced and the following relationships established:

British System
1 cubic yard (cu. yd.) = 27 cubic feet (cu. ft.)
1 cubic foot (cu. ft.) = 1728 cubic inches (cu. in.)

Metric System
1 litre (l) = 1000 cubic centimetres (cu. cm.)
1 litre = 1000 millilitres (ml.)

With the use of one-inch cubes students then develop the formula V = lwh for finding the volume of a rectangular solid where "V" represents the measure of the volume, "l" represents the length, and "w" represents the width, and "h" represents the height.

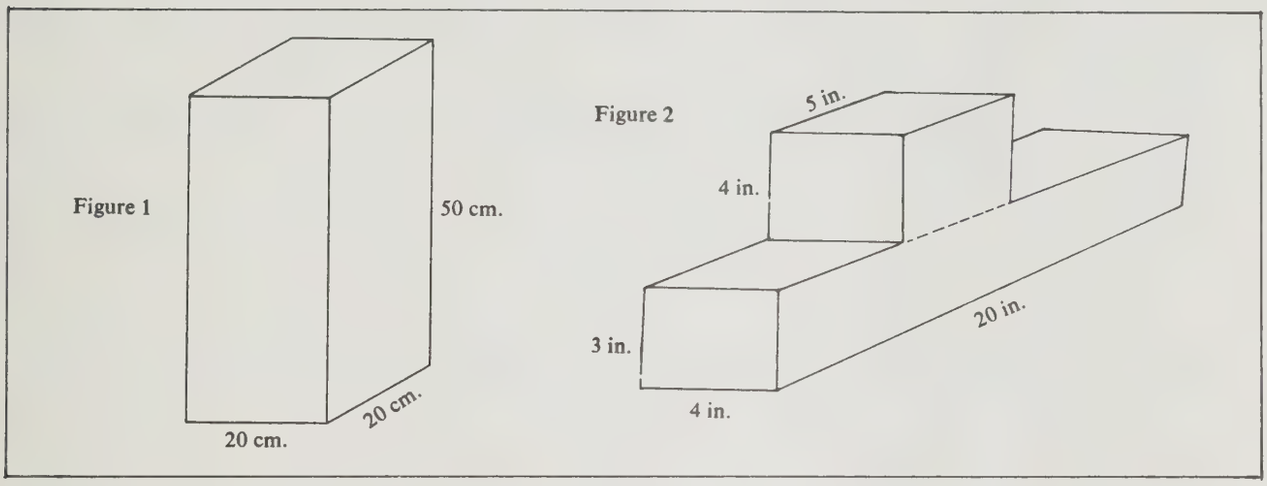
The program closes as several questions involving volume are assigned and the solutions are discussed.

PREPARATION FOR TELECAST

Students need the usual geometric instruments and workbooks in order to participate in the program. If full-sized models of the cubic inch, cubic foot, cubic yard, cubic centimetre and litre are available, they will help the students to obtain a practical knowledge of these units. Such commodities as water, sand, wooden cubes, and containers of different sizes and shapes would be useful for the post-viewing activities.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify any difficulties arising from it.
2. Students could be encouraged to estimate the volume of different objects and then be given the opportunity to decide methods of comparing, measuring and calculating their volumes. Water, sand and wooden cubes may serve as useful aids in developing fundamental ideas associated with volume.
3. Questions such as the following could be assigned in order to give students practice in calculating volume:
   a. Find the volume of water required to fill the container illustrated in figure 1. Give your answer to the nearest litre.
   b. Find the volume of the wooden model illustrated in figure 2.



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program seventeen

VOLUME OF PRISMS AND CYLINDERS

With the use of concrete materials, this program develops the idea that the volume of a right prism and cylinder may be found by multiplying the area of the base by the respective height.

PROGRAM CONTENT

The program opens by reviewing the formula V = lwh for finding the volume of a rectangular prism. However, by observing models of rectangular prisms made from inch cubes, it is noted that the volume can also be found by multiplying the area of the base by the height. Thus a formula V = Bh is established for finding the volume of a rectangular solid, where "V" represents the measure of the volume; "B" represents the area of each base and "h" represents the height. (Note that the height is the perpendicular distance between the two bases of a prism.) In this case, the formulas V = lwh and V = Bh are comparable, since "lw" in the first represents the area of the base, as does "B" in the second. Through discussion and the use of the concrete models, it is then established that the formula V = Bh can be used to calculate the volume of other prisms and cylinders where no length or width is involved.

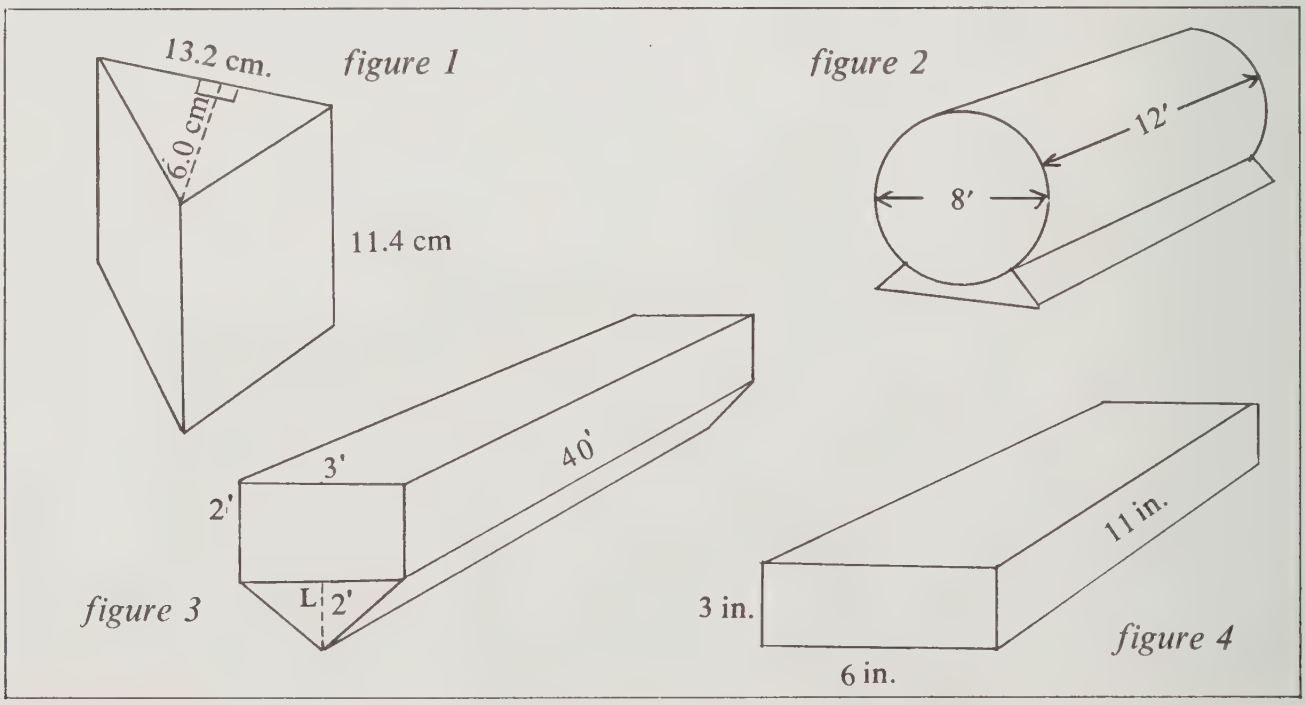
Several questions concerning the volume of prisms and cylinders are assigned and their solutions discussed.

PREPARATION FOR TELECAST

The usual geometric instruments and workbooks are needed to assure student participation in the program. The concrete materials which were suggested for program sixteen will also be useful for the post-viewing activities in this program.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.
2. Students could be encouraged to devise means of finding the volumes of prisms and cylinders as well as other shapes, such as cones, spheres and irregular objects.
3. Questions such as the following could be assigned in order to give students practice in calculating volume:
   a. Find the volume of the water required to fill a circular swimming pool to a depth of 3 feet if the diameter of the pool is 10 feet. Give your answer to the nearest cubic foot.
   b. Find the volume of the triangular prism which is illustrated in figure 1. Give your answer to the nearest cubic centimetre.
   c. The diagram in figure 2 represents a steel tank on a water truck. Find, to the nearest gallon, how many gallons of water it will hold if each cubic foot equals $6\frac{1}{4}$ gallons. Use $\pi \doteq \frac{22}{7}$
   d. Find the amount of earth in cubic yards removed in digging a trench with the shape and dimensions given in figure 3.
   e. A block of metal with dimensions given in figure 4 is to be made into a cylindrical bar with radius of 3 inches. Calculate to the nearest foot the length of the bar formed. Use $\pi \doteq \frac{22}{7}$



A series of 5 programs on Statistics and Probability

program eighteen

STATISTICS

The study of statistics is introduced in this program, which presents data through the pictograph and the bar graph.

PROGRAM CONTENT

The program opens with four students analysing their newspapers. They are discovering that numbers are used in reporting the news of their interest. Each student finds where the data of his interest originated; one of the boys takes us to a football field where he inquires how statistics are used, one of the girls takes us to a meteorological institute where the weather facts originate. A statistical expert from a stock brokerage firm is interviewed, and then, a home economist.

Data can be presented in many ways. In this program, pictographs are explained as a useful means of picturing numerical relationships. Horizontal and vertical bar graphs are introduced and we see how they clearly and quickly summarize data relationships.

Students become aware of the wide application of statistics in our modern scientific age.

PREPARATION FOR TELECAST

Each pupil will require the following materials: pencil and ruler, two sheets of graph paper, and two sheets of blank paper. It would be advisable for the teacher to tell the pupils in advance that they will be drawing a bar graph during the program.

SUGGESTED PRE-VIEWING ACTIVITIES

1. A class discussion of the statistics which confront us daily in the newspapers, magazines, and other forms of communication would be useful.
2. A review of "rounding off" numbers might be helpful.
3. Before the telecast, the pupils might be asked the following question, which will be answered on the program: "In what ways do we collect and use data?"

SUGGESTED POST-VIEWING ACTIVITIES

1. Data for the construction of a bar graph is given at the end of the program. You may wish to copy this data on the board if the students have not been able to complete copying it from the screen.

   Data for the Bar Graph

   Title: Population of five Ontario Metropolitan Areas (Census 1961)

CITY	POPULATION
Hamilton	365,189
Kitchener	153,864
Windsor	193,365
Sudbury	110,694
Ottawa	429,750

Suggestion: Round off population figures to the nearest ten thousand.

2. Another suggested exercise in the program is the construction of a pictograph.

   Data for Pictograph

   Title: Dropouts Through the Grades

GRADE	YEAR	NUMBER
9	1961	100
10	1962	83
11	1963	62
12	1964	57
13	1965	33
University	1966	16

Suggestion: Have the pupils use a stick symbol *to represent a given number of students.*

3. The students could be encouraged to find examples of data, pictographs, and bar graphs in magazines and newspapers to make a classroom display.
4. A visit to a stock brokerage firm would give reality to the data found on the financial page of the local newspaper.

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program nineteen

STATISTICS

The study of statistics is further developed with the demonstration of how to draw and use a circle graph, a frequency distribution, a histogram, and a line graph.

PROGRAM CONTENT

At the beginning of the program the bar graph that the students were left to draw as a follow-up for program 18 is shown. The rectangular distribution graph leads to a detailed outline of how to make a circle graph, accompanied by a few examples of its use.

Frequency distribution is clarified by familiar examples, such as counting the frequency of different types of vehicles in traffic. Tallies are used to determine the frequency, and this data is organized in frequency distribution tables. Histograms are developed from these tables; mark intervals and group boundaries are discussed in this context.

MARK INTERVAL	TALLY	FREQUENCY
28.5 to 30.5	//	2
26.5 to 28.5	𝍸	5
24.5 to 26.5	𝍸 𝍸 /	11
22.5 to 24.5	𝍸 ////	9
20.5 to 22.5	𝍸	5
18.5 to 20.5	///	3

Several types of line graphs exist. Among those discussed on the program are the frequency polygon, and straight line to show rate and comparison.

PREPARATION FOR THE TELECAST

Each pupil will require the following materials: pencil and ruler, two sheets of blank paper, two sheets of graph paper, and the completed assignment from program 18.

SUGGESTED PRE-VIEWING ACTIVITIES

1. As an introduction to the construction of circle graphs:
   a. Review the use of the protractor to construct angles of given sizes.
   b. Review the measurement of angle formed by two radii of a circle.
2. Review line graphs showing rate and comparison.

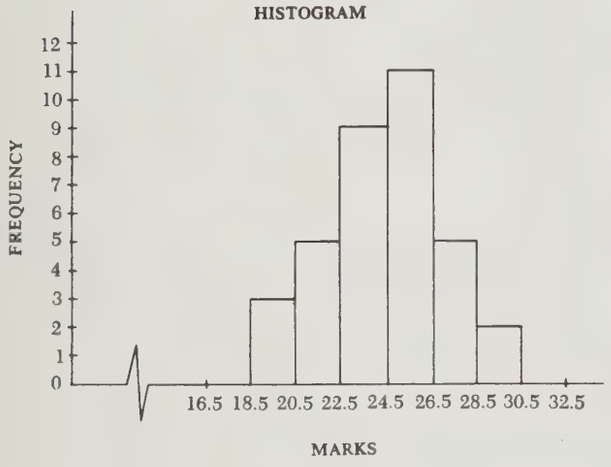
SUGGESTED POST-VIEWING ACTIVITIES

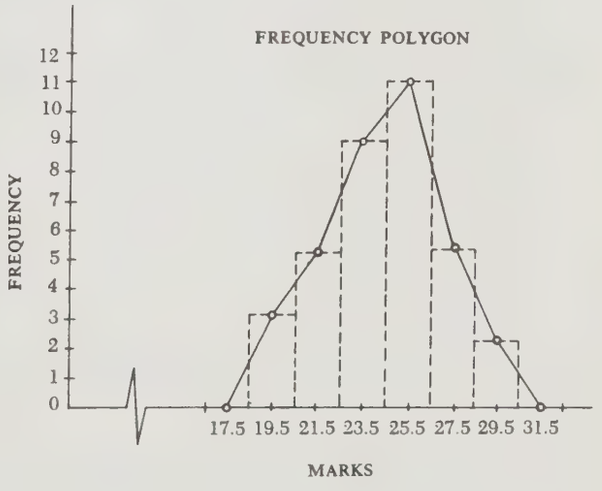
1. On the program, the pupils will be given 35 test scores, from which they are requested to construct a combined histogram and frequency polygon for the distribution. Unfortunately, time does not allow these numbers to be held on the screen long enough for all pupils to copy them. We suggest that the teacher copy the list of figures on the board following the program, as well as the accompanying instructions.

DATA: 35 TEST SCORES

21	26	21	20	23	24	22
19	24	26	25	23	26	29
21	24	19	25	26	25	22
25	27	23	26	24	25	30
25	23	27	24	28	28	28

The following diagrams illustrate the answers which will also be shown at the beginning of Program 20.





program twenty

STATISTICS: MEASURES OF CENTRAL TENDENCY

This program familiarizes the student with three different measures of central tendency: the arithmetic mean, the median, and the mode.

PROGRAM CONTENT

The program begins with the answer to the question given at the end of program 19. Following this is a brief review of the various types of graphs which have been studied in the previous two programs.

"Average" is introduced as another way of summarizing a set of data, in this case, by means of a single score. Different meanings of the word "average" are discussed; experiments and data which are of interest to the students illustrate the true meanings of mode, median, and arithmetic mean.

PREPARATION FOR TELECAST

Each student will require the following materials: pencil and ruler, two sheets of blank paper, and the completed assignment from program 19.

SUGGESTED PRE-VIEWING ACTIVITIES

1. The class might discuss the meaning of the word "average" as it exists in the mind of the pupils.
2. Review briefly the frequency distribution chart, and the following graphs: pictograph, bar graph, circle graph, histogram, and line graph.

SUGGESTED POST-VIEWING ACTIVITIES

1. The program presents the following numbers for the students to analyse after the program. They are asked to find the mode, the median, and the mean of the numbers: 2, 3, 4, 7, 5, 7, 3, 3, 10, 4, 9. The answers to this question will not be dealt with on program 21. They are included here for classroom use.
   a. Mode: 3 (because it appears most often or has the greatest frequency.)
   b. Median: First the numbers must be arranged in order: 2, 3, 3, 3, 4, 4, 5, 7, 7, 9, 10. The median is 4.
   c. Mean: First the total must be calculated, and then divided by the number of numbers involved:

$$\frac{2+3+4+7+5+7+3+3+10+4+9}{11} = \frac{57}{11} = 5\frac{2}{11}$$ The mean is $5\frac{2}{11}$.

2. Another exercise in central tendency is suggested. Find the mode, median and mean of the following data: 3, 3, 1, 2, 2, 3, 2, 4, 2, 3, 3, 4.

   Answers: a. Mode 3
   b. Median 3
   c. Mean 3

program twenty-one

PROBABILITY

This program endeavors to develop the concept of probability as the ratio of the number of favorable cases to the total number of cases.

$$Probability = \frac{number\ of\ favorable\ cases}{total\ number\ of\ cases}$$

PROGRAM CONTENT

The familiarity of "probability" statements in conversation is illustrated in the introduction to this program. Probabilities are calculated in simple cases where the number of favorable and total cases can easily be discovered, for example: the probability of getting a head when flipping a coin; of a five when rolling dice.

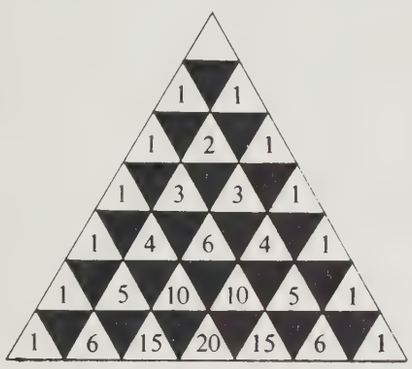


DIAGRAM OF PASCAL'S TRIANGLE

Probability charts which list the possible outcome when one flips two coins, three coins, and so on are described. These charts introduce "Pascal's Triangle", which is used to determine probabilities.

The probability board is another method of predicting probabilities.

PREPARATION FOR TELECAST

Each pupil will require the following materials: pencil, ruler and two sheets of blank paper.

SUGGESTED PRE-VIEWING ACTIVITIES

1. Review briefly the unit of Curriculum 1.12B dealing with Ratios.
2. Review the five regular solids that were presented on a previous telecast. These are the tetrahedron, hexahedron, octahedron, dodehedron and icosahedron.
3. Practice in reducing fractions to their simplest form.

SUGGESTED POST-VIEWING ACTIVITIES

1. The class might construct a probability board.
2. Discuss all the possible outcomes when two dice are rolled, and have the students set up a chart.
3. Pupils might write biographical sketches of famous mathematicians such as Blaise Pascal and Pierre LaPlace.
4. Discuss the probabilities of each of the following events:
   a. Tossing the sum of 7 with 2 dice, 3 dice, etc
   b. Obtaining an ace from a deck of cards

program twenty-two

PROBABILITY

Probability is presented in this program as an exciting and rapidly growing branch of mathematics.

PROGRAM CONTENT

The program begins with a brief review of program 21.

In studying probability, when the total number of cases cannot be counted, one is faced with the "need for sampling". An instance of this occurs in television ratings, which are based on such samples.

The meaning of "odds" is distinguished from the meaning of "probability". The law of large numbers is introduced, which states that as the number of trials is increased, the calculated probability approaches the value of the "Mathematical Probability Ratio". This is illustrated by tossing a coin many times. "Empirical Probability" is explained as it is used in industry, commerce and sports.

The program closes with "doop"—the game of predicting "doubles".

PREPARATION FOR TELECAST

Each pupil will require the following materials: pencil and ruler, two sheets of blank paper.

SUGGESTED PRE-VIEWING ACTIVITIES

1. Multiplication of decimal fractions might be reviewed.
2. Review division by a common fraction using the reciprocal method.
3. The students should be familiar with "Probability Ratio", which was introduced on program 21.

$$\text{Probability} = \frac{\text{number of favorable cases}}{\text{total number of cases}}$$

SUGGESTED POST-VIEWING ACTIVITIES

1. Have each pupil in the class flip a coin ten times. Record the combined results, and calculate the probability of a "head". Compare this with the mathematical probability of ½.
2. A bowl contains three red, two blue, three yellow and two green marbles. A single marble is drawn from the bowl.
   a. What is the probability that it is red?
   b. What are the odds in favor of its being red?
   c. What is the probability that it is blue or green?
   d. What are the odds against its being blue or green?
   e. What is the probability that it is yellow?

The answers are as follows:

a. P (red) $= \frac{3}{10}$

b. Odds in favour of red $= \frac{\frac{3}{10}}{\frac{7}{10}} = \frac{3}{7}$

c. P (blue or green) $= \frac{4}{10}$ or $\frac{2}{5}$

d. Odds against blue or green $= \frac{\frac{6}{10}}{\frac{4}{10}} = \frac{6}{4} = \frac{3}{2}$

e. P (yellow) $= \frac{3}{10}$

3. The following table which gives the probability of two persons in a group having the same birthday could be discussed.

Number of persons in the group	*Probability of two persons having same birth date*
5	.03
10	.12
15	.25
20	.41
25	.57
30	.71
40	.89

FOR OPTIMUM VIEWING CONDITIONS

1. Review Teacher's Guide well in advance of each telecast.

2. Have the set on and adjusted, sound off, prior to telecast.

3. Assure student comfort
   - — located for best viewing
   - — no glare, reflection on screen
   - — room properly ventilated
   - — suggested materials ready
   - — within maximum of 20′ of the screen

4. Seat yourself where you can observe both the telecast and your students' reactions.

This diagram shows the suggested placement of the television set: